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# PARAMETERS AFFECTING THE MEASUREMENT OF AERO ENGINE EXHAUST SMOKE

A Statistical Analysis of Test Data

DONALD L. CHAMPAGNE, FIRST LIEUTENANT, USAF

TECHNICAL REPORT AFAPL-TR-70-23
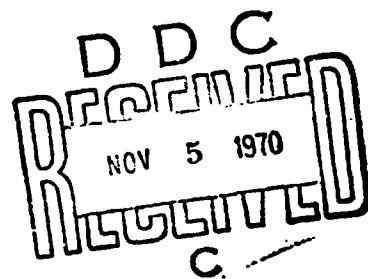
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AIR FORCE AERO PROPULSION LABORATORY
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WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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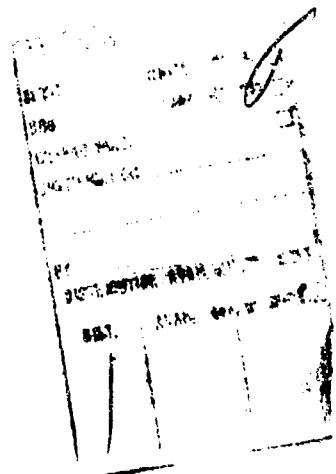
PARAMETERS AFFECTING THE MEASUREMENT OF AERO ENGINE EXHAUST SMOKE

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FOREWORD

This report was prepared by the Fuel Branch of the Fuel , Lubrication, and Hazards Division, Air Force Aero Propulsion Laboratory, under Project 3048, Task 304805.

The experimental data used as a basis for this report is from tests con-
ducted by the Society of Automotive Engineers Technical Committee E-^ in June 1969. Raw data was reduced to the final form presented h e r e i n by a group within the SAE Committee and by a team at General Electric Company, Evendale, Ohio.

Some of the items compared in this report were commerical items that were not developed or manufactured to meet Government specifications and were not necessarily intended for the service considered in this report. Any failure to meet the objectives of this study is no reflection on the value of these items for other service, nor should the conclusions of this report be construed as statements of the manufacturers' abilities.

The analysis described in this report was conducted from September 1969 to February 1970 at the Air Force Aero P r o p u l s i o n Laboratory, Wright-Patterson AFB , Ohio 45433.

The author appreciates and acknowledges the assistance rendered by the following outside of the AF Aero Propulsion Laborrtory and the SAE Com-mittee: Mr. C. Fetter of the Digital Computation Directorate, Aeronautical Systems Division, for guidance in applying the data plotting routine "GP;" Mrs. Mary Lum of the Operations Analysis Office, AF Logistics Command, for reviewing and commenting on the approach and on the analysis criteria; and Mr. Charles Stanforth and others at the General Electric Company for their help in reducing the data to a final form for analysis.

This report was submitted by the author 23 March 1970.

This technical report has been reviewed and is approved.

Arthur V. Churchill
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Air Force Aero Propulsion Laboratory

ABSTRACT

This report describes a computerized statistical analysis of test data from engine smoke measurements conducted by the Society of Automotive Engineers Technical Committee E-31. This Committee was organized to develop a reasonably simple, precise, and universally acceptable standard for measuring exhaust smoke from aircraft engines. The analysis indicated that the Committee's test data can be used to arrive at statistically meaningful conclusions about four measuring system parameters. "Whatman No. 4" was found to be superior to "Millipore SM" as a filtering medium in this application. All three reflectometers tested were found to produce equivalent results. White reflectometer background shade was found to have slight superiority over black, yet black (i.e., absolute reflectance less than 5%) was recommended as a safeguard against unknown factors. The lower sampling flow rate (0.0041 scfs) was found to have produced slightly, yet consistently, higher smoke density readings than the higher flow rate (0.0085 scfs) tested.

AFAPL-TR-70-23

TABLE OF CONTENTS

ILLUSTRATIONS

TABLES

SECTION I

INTRODUCTION

There are numerous systems in use for measuring exhaust smoke from aircraft engines. Most consist of drawing an exhaust smoke sample through a filter, measuring the light reflection from the resultant spot, and then comparing this to the light reflection from some standard.

Unfortunately, the many details of this seemingly simple procedure have never been standardized. Results from different systems are not readily comparable, and the inherent precision of most of these systems has never been defined.

Technical Committee E-31 of the Society of Automotive Engineers (SAE) was established to cope with this problem. Its purpose was to prepare a reasonably simple, precise, and universally acceptable method for measuring exhaust smoke from aircraft engines.

In June 1969, Committee E-31 compounded a preliminary standard and conducted tests to examine the parameters of this proposed scheme. A brief description of the test program and the procedure evaluated are given in Appendix I.

"SN" is the dimensionless term proposed for use in quantifying smoke emission. Some of the test program raw data was reduced to SN by an analysis group within the Committee. Additional data was later reduced by a team at General Electric Company (GE), Evendale, Ohio. The data reduction procedures of the two groups differed somewhat. Both are described in Appendix II.

An analysis of this reduced test data is presented in this report.

This analysis of reduced data was undertaken to answer the following:

How much did each parameter influence the measurement of smoke in comparison to all other parameters investigated?

If a parameter did have effect, which value of the parameter produced the best results?

1

SECTION II

ANALYSIS PROCEDURE

Table I contains all reduced (SN) data, the raw material of this analysis. Each data column contains data for a combination of four explicit parameters:

- **Filter Medium:** Whatman No. 4 or Millipore SM, plain white

- **Reflectometer:** MacBeth Model NB-100R; W. W. Welch "Densichron," Model One; or Photovolt Model 610

- **Sampling Flow Rate:** 0.0041 standard cubic feet per second (scfs) or 0.0085 scfs

- **Reflectometer Background Shade:** Black or white

The group which reduced each data column (SAE or GE) is also noted in Table I.

The rows of Table I are numbered 15 through 56 in keeping with the numbering system established during the tests. Each of these 42 rows represents different engine conditions coupled with values of parameters other than the four noted above. (Appendix I contains a complete list of parameters.) This is an important point that largely dictated the analysis method: more than four parameters were varied during the tests. Consequently, it is not possible to make column comparisons unless all columns being compared contain exactly the same rows, not just the same number of rows.

Initially it seemed possible to draw statistically valid conclusions about six parameters. More detailed scrutiny revealed that this unfortunately was not possible. There was not enough data to statistically examine any parameter other than the four explicitly noted as column headings in Table I.

Column "sets" were established to overcome the lack of identical test conditions from row to row. A set is any number of columns all of which contain the same rows. Since all the columns of Table I do not all include the same rows, forming a set was necessarily a compromise between getting as many points per column as possible, while including as many columns as possible in the set. For example, see Tables II through VI.

2

TABLE I REDUCED DATA (SN) FOR ANALYSIS

Row No	WHATMAN																		MILLIPORE																		
	Mac Beth				Densichron				Photovolt								MacBeth				Densichron				Photovolt													
	0085		0041		0085		0041		0085				0041				0085		0041		0085		0041		0085				0041									
	B	W	B	W	B	W	B	W	B		W		B			W	B	W	B	W	B	W	B	W	B		W		B		W							
	SAE				GE				SAE	GE	SAE	GE	SAE	GE	SAE	GE	SAE				GE				SAE	GE	SAE	GE	SAE	GE	SAE	GE						

(Table data largely illegible due to scan quality; row numbers run from 15 through 56.)

The following four space code was devised to identify the parameter values of any column:

● First space identifies filter medium

    W - Whatman

    M - Millipore

● Second space identifies reflectometer

    M - MacBeth

    D - Densichron

    P - Photovolt

● Third space identifies sampling flow rate

    4 - 0.0041 scfs

    8 - 0.0085 scfs

● Fourth space identifies reflectometer background

    B - black

    W - white

For example, WM8B is a column containing reduced data taken on <u>Whatman</u> filter medium, at the high (<u>0.0085 scfs</u>) flow rate, with the resultant spot read with the <u>MacBeth</u> meter using the <u>black</u> background. Note that this code does not reveal which or how many rows are included.

Each of the four parameters was analyzed independently of the other three. All the column sets for one parameter constitute a "series." No one of the four series contained all the data of Table I, but each utilized at least 90% of that reduced data.

The computer routine CORRE1, included as Appendix III, was written around two existing subroutines for this analysis. Figure 1 is a typical print-out of this program for a single column set. The computer outputted all input data, made correlation plots (scatter diagrams), and computed the following:

● Mean (M) of each column

● Standard deviation (SD) of each column

- Standardized mean and standardized standard deviation for each column

- Coefficient of variation (CV) of each column
- Correlation coefficient (r) of each column pair specified

These quantities are defined and explained in Appendix IV.

There was no preanalysis attempt to correct or exclude suspect data.

INPUT DATA

WKBB	WKBW	WKBB	WKBW	MKBB	MKBW	MKBB	MKBW
46.00	54.00	45.00	51.60	46.00	48.00	45.50	46.90
58.00	64.00	55.10	60.60	58.00	59.00	58.90	59.90
51.00	58.00	50.50	56.30	57.00	57.00	57.40	58.60
29.00	37.00	26.20	33.90	20.00	18.00	23.10	38.20
58.00	65.00	60.80	65.80	59.00	59.00	60.60	61.40
30.00	40.00	28.30	35.60	25.00	23.00	25.50	26.60
10.00	18.50	11.80	16.70	8.00	7.00	9.00	9.80
49.00	57.00	46.80	53.30	47.00	49.00	49.30	50.40
28.00	36.00	27.00	34.50	24.00	24.00	25.90	27.50
45.00	55.00	45.60	51.20	44.00	47.00	43.60	44.60
29.00	31.00	28.70	35.50	20.00	21.00	23.00	23.70
6.00	10.00	5.60	8.00	5.00	4.00	4.10	4.50
4.50	10.00	5.60	7.90	6.00	2.00	3.70	4.10
19.00	29.00	17.40	24.30	12.00	10.00	14.70	15.50
33.00	44.00	30.00	37.30	26.00	24.00	28.30	29.50
38.00	49.00	36.60	44.10	36.00	36.00	35.60	37.50
18.00	27.00	17.30	24.30	14.00	14.00	15.20	16.20
32.00	42.00	33.20	37.60	20.00	22.00	23.00	24.20
18.00	27.00	16.30	22.90	14.00	15.00	14.70	15.20
33.00	44.00	31.40	38.40	29.00	28.00	29.1	30.50
14.00	21.00	13.50	18.80	11.00	10.00	10.50	11.30
28.00	38.00	27.60	34.70	24.00	24.00	27.70	24.70
50.00	58.00	46.80	53.10	50.00	52.00	51.20	52.40
56.00	52.00	52.90	59.10	58.00	59.00	61.00	61.30
18.00	26.00	14.20	19.70	7.00	8.00	8.40	10.10
36.00	42.00	27.20	36.10	26.00	22.00	23.00	24.30
56.00	60.00	46.90	52.50	47.00	50.00	43.50	44.50
66.00	67.00	63.30	63.10	56.00	56.00	52.70	53.30
63.00	62.00	60.60	60.80	47.00	52.00	55.40	55.80
46.00	51.00	42.30	47.70	49.00	51.00	49.00	50.60
49.00	56.00	42.90	50.50	55.00	55.00	57.70	55.90
17.00	25.00	25.90	24.10	10.00	9.00	9.30	10.30
28.00	59.00	43.30	41.20	20.00	23.00	23.10	24.50
51.00	59.00	58.10	58.50	43.00	46.00	43.10	44.60
58.00	63.00	62.00	61.30	57.00	56.00	51.60	53.00

THIS DATA IS FROM THE FOLLOWING ROWS, INCLUSIVE, READING FROM COLUMN (TOP TO BOTTOM) - 17-25,27-29,32-35,37-49,51-56

Figure 1. Typical Print-Out of Analysis Routine CORRE1 for One Set

6

AVERAGES, STANDARD DEVIATIONS AND COEFFICIENTS OF VARIATION

CELL (PARAMETER SET)	NUMBER OF DATA POINTS (SET POINTS)	ARITHMETIC MEAN (AMN)	STANDARD DEVIATION (SD)	STANDARDIZED ARITH MEAN (AMN/AAMN)	STANDARDIZED STAN DEVIATION (SD/ASD)	COEFFICIENTS OF VARIATION (CV) (100 X SD/AMN)
WH8B	35	36.300	17.304	1.007	0.978	47.671
WH8W	35	43.614	16.060	1.210	0.908	36.822
WD8B	35	35.620	16.939	0.988	0.958	47.555
WD8W	35	40.600	16.347	1.126	0.924	40.264
MH8B	35	32.171	18.614	0.892	1.052	57.860
MH8W	35	32.571	19.426	0.903	1.098	59.641
MD8B	35	33.086	18.483	0.918	1.045	55.864
MD8W	35	34.486	18.342	0.956	1.037	53.186

AVERAGE ARITHMETIC MEAN (AAMN)   =   36.056
AVERAGE STANDARD DEVIATION (ASD)   =   17.689

CORRELATION COEFFICIENT
BETWEEN X AND Y

X	Y	
WH8B	WH8W	0.98194
WD8B	WD8W	0.98475
MH8B	MH8W	0.99610
MD8B	MD8W	0.99106

PLOTTING OF CORRELATED COLUMN PAIR

PERFECT CORRELATION LINE IS DRAWN WITH DOTS
ACTUAL COLUMNS PLOTTING IS WITH CROSSES
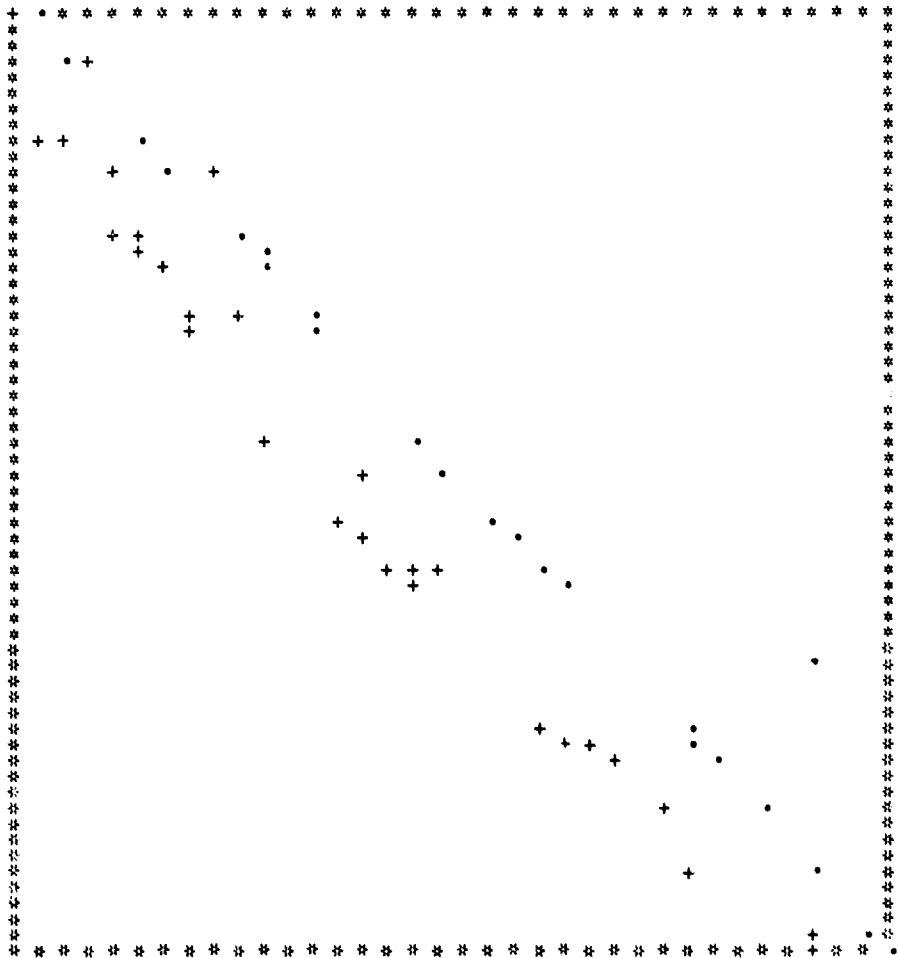
XMIN = 0.45000000E 01   XMAX = 0.66000000E 02   YMIN = 0.45000000E 01   YMAX = 0.66999999E 02

ABSCISSA (X) IS WM8R
ORDINATE (Y) IS WN8W
CORRELATION COEFFICIENT BETWEEN X AND Y = 0.98194

8

PLOTTING OF CORRELATED COLUMN PAIR

PERFECT CORRELATION LINE IS DRAWN WITH DOTS
ACTUAL COLUMNS PLOTTING IS WITH CROSSES
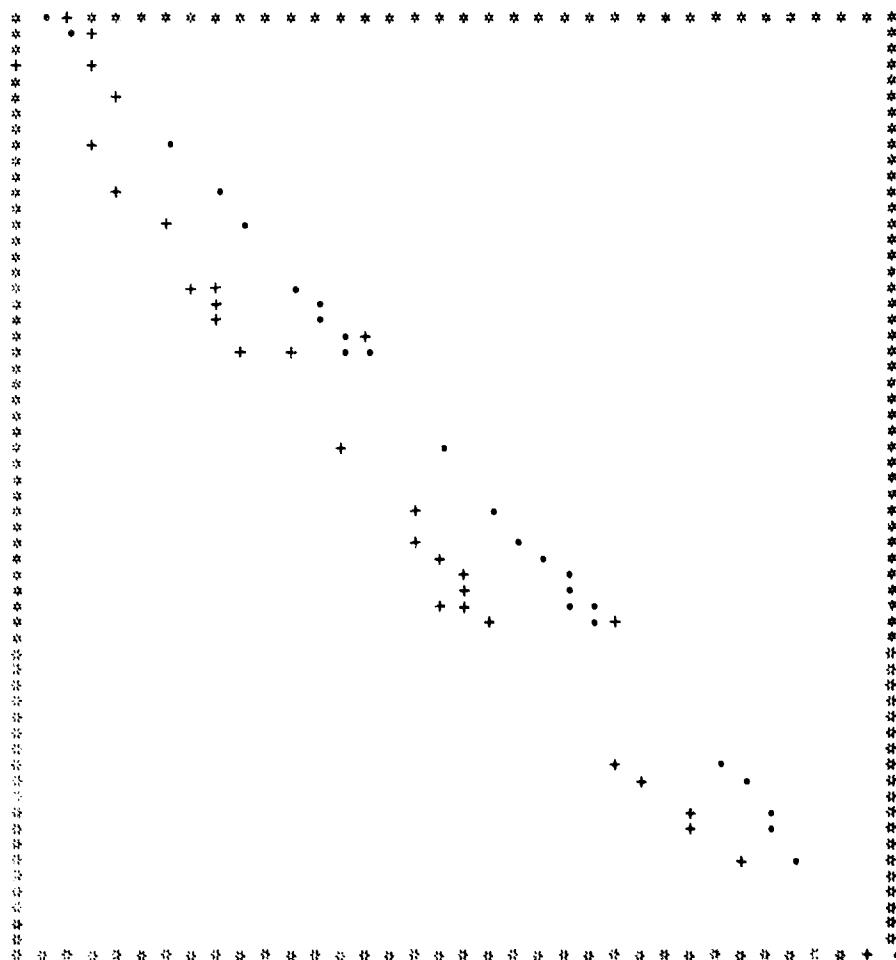
XMIN = 0.5599999E 01   XMAX = 0.6329999E 02   YMIN = 0.5599999E 01   YMAX = 0.6579999E 02

ABSCISSA (X) IS WD8B
ORDINATE (Y) IS WD8W
CORRELATION COEFFICIENT BETWEEN X AND Y =   0.98475

PLOTTING OF CORRELATED COLUMN PAIR

PERFECT CORRELATION LINE IS DRAWN WITH DOTS
ACTUAL COLUMNS PLOTTING IS WITH CROSSES
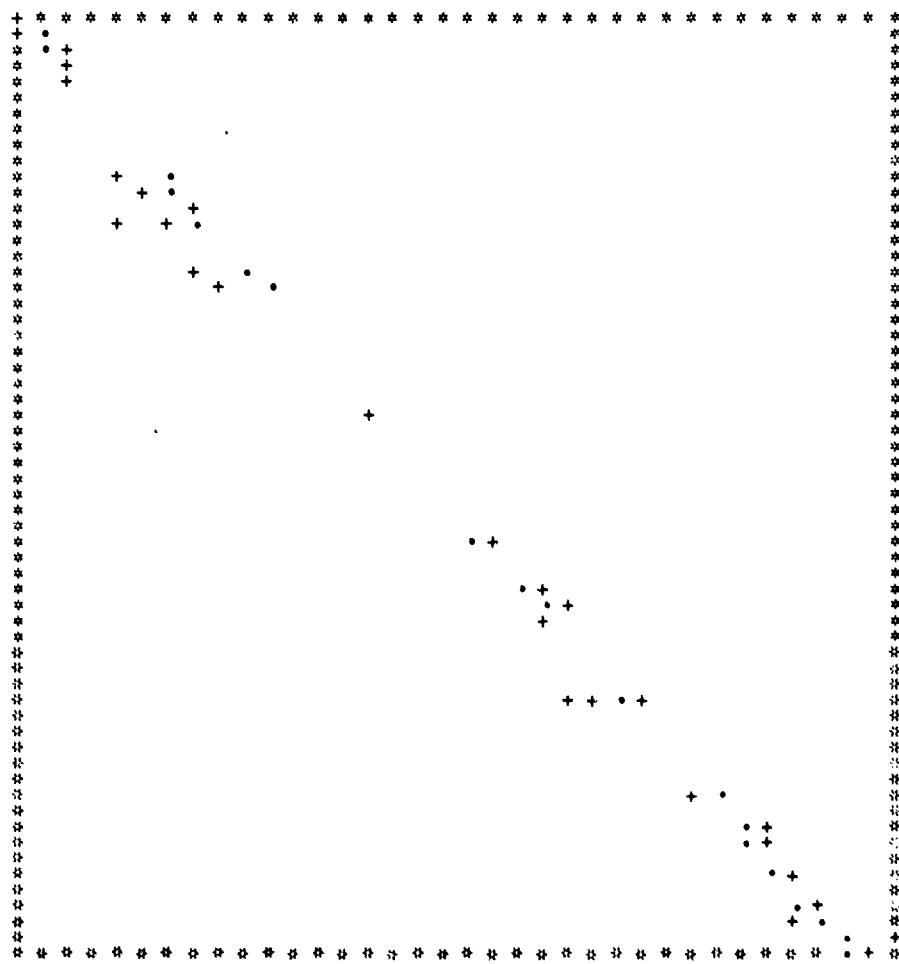
XMIN = 0.49999999E 01   XMAX = 0.59000000E 02   YMIN = 0.20000000E 01   YMAX = 0.59000000E 02

ABSCISSA (X) IS MM8B
ORDINATE (Y) IS MM8W
CORRELATION COEFFICIENT BETWEEN X AND Y =   0.99610

10

PLOTTING OF CORRELATED COLUMN PAIR

PERFECT CORRELATION LINE IS DRAWN WITH DOTS
ACTUAL COLUMNS PLOTTING IS WITH CROSSES

ABSCISSA (X) IS MD8B
ORDINATE (Y) IS MD8W
CORRELATION COEFFICIENT BETWEEN X AND Y = 0.99106
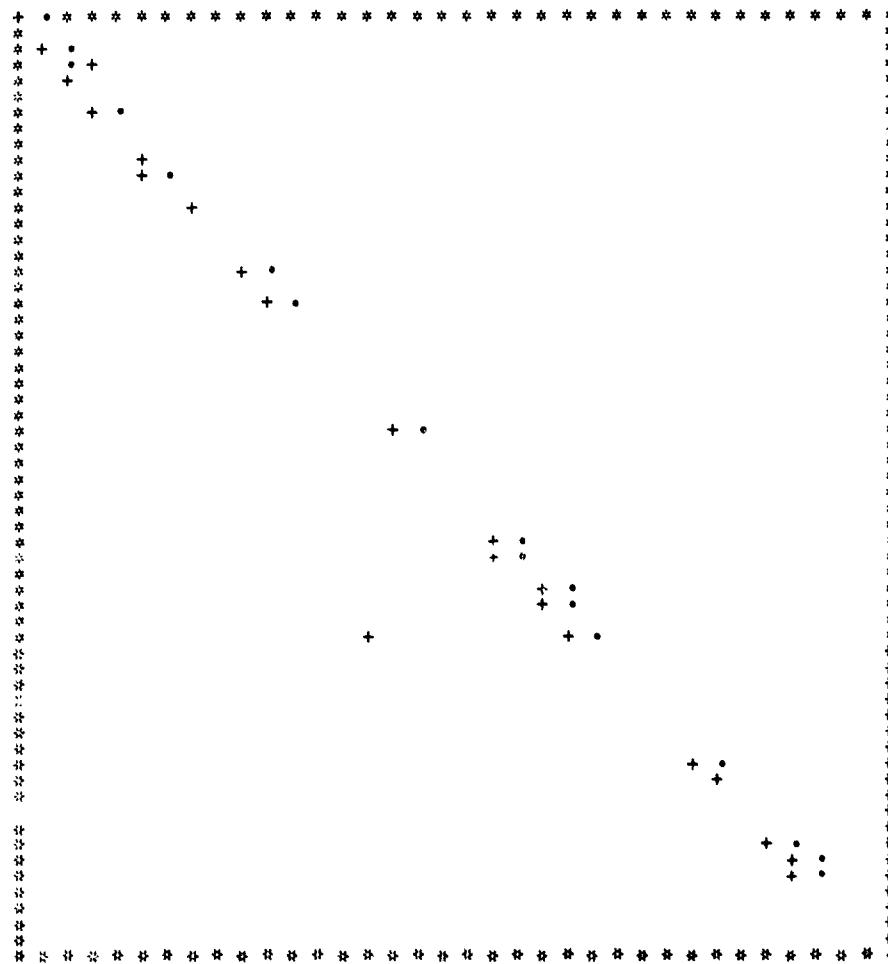
XMAX = 0.61000000E 02   YMIN = 0.37000000E 01   YMAX = 0.61499999E 02

XMIN = 0.37000000E 01

SECTION III

RESULTS

1. GENERAL CONSIDERATIONS

The results of this analysis are tabulated in Tables II through VI.

The number of data points per column is the most important statistical indicator of confidence. About 15 points per column was generally the minimum number that produced good results.

The data had to be examined closely after results were c o m p u t e d. In several cases, what appeared to be poor results was actually attributable to just a few "odd points," that is, deviations from whatever trend was established by the rest of the data in a column pair. In such cases, these few points were corrected to a value that seemed probable, and the results were recomputed. Tables II through VI contain only results computed with uncorrected data. Any corrected results are noted and listed in the "Comments" column of each Table.

Comparisons can be made within column sets only. This, as stated previously, is because of the lack of identical test conditions from set to set. Even though two columns may bear the same column identification codes, they are generally not identical if they appear in different sets; the rows and number of rows comprising each set are different.

2. ANALYSIS CRITERIA

Two types of criteria were used to meet the objectives of this analysis. Influence criteria were used to determine how much, if any, influence each parameter had on the measurement of smoke. Superiority criteria were subsequently used to determine which, if any, value of a given parameter produced better results.

The criteria for a parameter to have had significant influence were:

- ΔM (difference in column means) $> 10\%$ of the lower M
- ΔCV (difference in column coefficients of variation) $> 10\%$ of the lower CV

12

● $r < 0.990$, for a nontranslated function

Anyone of these had to be satisfied for a parameter to be considered as having significant influence. These distinction criteria are subjective. They were based on the author's preliminary survey of the computer computations and on the belief of several Committee E-31 members experienced in smoke measurement that the level of significance of the system's results was about 3 SN in 30.

It was important to qualify the $r < 0.990$ criterion as being valid only for a "nontranslated function." The definition of r considers dispersion of data, as well as deviation in slope of the data regression line from the slope of the perfect correlation line $y = x$. However, r does not consider the effect of a translated function $y = x + k$. Figure 2 shows a perfectly translated function. The correlation coefficient for both it and the perfect correlation line is 1.0. This translation phenomenon appeared fairly frequently in the correlation plots.

Two superiority criteria were used to distinguish between values of a given parameter. The best parameter value was the one that displayed:

● The largest M
● The smallest CV

These criteria are desirable from purely mathematical considerations of precision. They are also desirable criteria considering the nature of smoke measurement and the definition of SN. The SN scale is mathematically defined from 0 to 100. When smoke spots are rated in units of optical density, SN values are most precise at the scale midpoint, SN = 50. This is because the expression for SN in terms of optical density is a logarithmic function. Also, the need for precision in smoke measurement is greatest at that value of SN corresponding to the threshold of smoke visibility. Though this value is far from being well defined, all work to date indicates that it is within SN of 20 to 35 (References 1 through 3). Consequently, the best value of a parameter is not only the one that produces the least deviation with respect to the mean (minimizes CV), but the one that tends to increase the mean toward SN = 50.
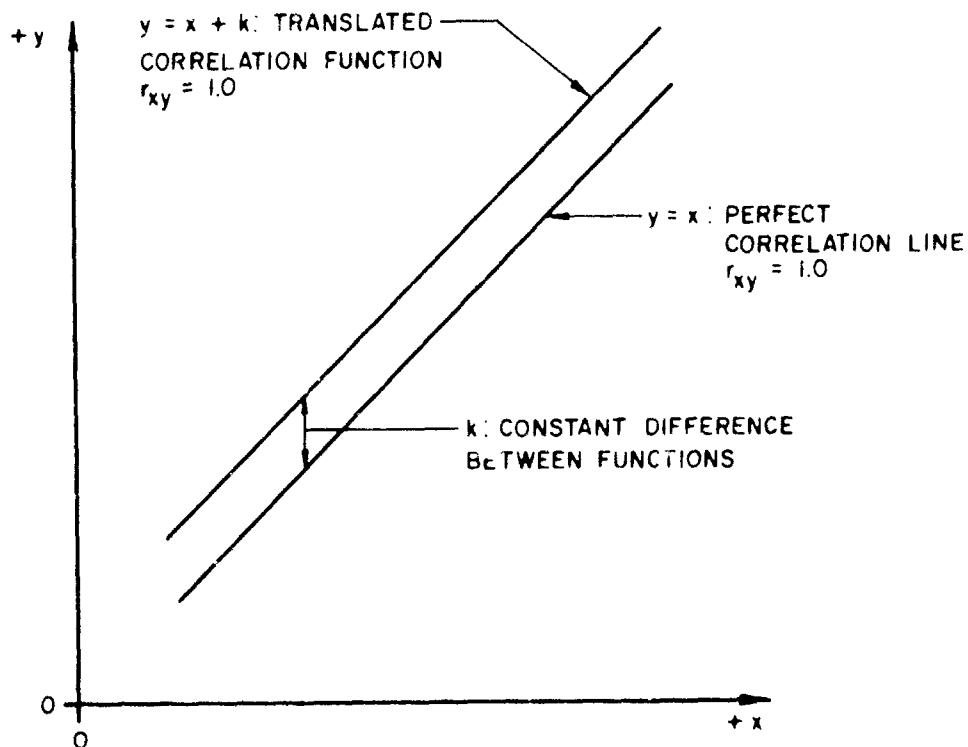
Figure 2. Typical Perfectly Translated Correlation Function

3. USE OF DATA REDUCED BY GE VS SAE GROUPS

First it was necessary to determine if the use of two data reduction groups with somewhat different methods had significantly influenced the results. Since both groups prepared what should ideally have been identical data with the Photovolt reflectometer, these data were used to investigate the possibility of influence. Table II contains the results of this comparison.

Table II contains four column sets, each with one column pair. ΔM was insignificantly small with all four sets, but ΔCV appeared to be significantly large in Set 2, and r appeared to be significantly low in Sets 1 and 2.

However, the correlation plots revealed that three of the 11 points per column in Set 2 were odd. Set 1 also displayed 3 odd points in its correlation plot of 22 points. Correction of these odd points made both suspect r greater than 0.990, and reduced the ΔCV of Set 2 to below 4.2.

14

TABLE II RESULTS OF GE VS SAE REDUCED DATA COMPARISON

DATA COLUMN IDENTIFICATION CODE	NUMBER OF POINTS PER COLUMN/ROWS INCLUSIVE FROM TABLE I	MEAN (M)	DIFFERENCE IN MEANS (ΔM)	COEFFICIENT OF VARIATION (CV)	DIFFERENCE IN COEFFICIENT OF VARIATION (ΔCV)	CORRELATION COEFFICIENT (r)	COMMENTS
SET 1 WP88(GE) WP88(SAE)	22 /15-18, 32-34, 37-40, 45-49, AND 51-56	32.5 30.8	1.6	49.5 51.3	1.8	0.989*	LOW r ATTRIBUTABLE TO 3 ODD POINTS IN PAIR OF 22.
SET 2 WP48(GE) WP48(SAE)	11 /15-18, 34, 47, 48, AND 53-56	40.7 38.2	2.5	42.2 46.9	4.7*	0.987*	LOW r AND HIGH ΔCV ATTRIBUTABLE TO 3 ODD POINTS OF 11 TOTAL.
SET 3 WP88(GE) WP88(SAE)	10 /32-40, 45-48, AND 52-56	27.6 27.0	0.6	59.1 57.8	1.3	0.996	NO ABERRATIONS NOTICED. THIS IS THE BEST SET IN THE SERIES.
SET 4 MP48(GE) MP48(SAE)	7 /34, 45, 46, AND 53-56	27.6 28.7	1.1	59.5 64.9	5.4	0.995	NO ABERRATIONS NOTICED.

* SIGNIFICANT DISTINCTION IN THE UNCORRECTED DATA. SEE CRITERIA IN SECTION III PARAGRAPH 2

It was concluded that no significant difference existed between the Photo-volt data reduced by GE and that reduced by SAE. For the purpose of this analysis, this conclusion was taken as general proof of the identity of results from the two data reduction methods.

The GE and the SAE Photovolt data were not mixed. The GE Photovolt data was chosen for the remainder of the analysis simply because that team had produced more points. The analysis was then based on all data in Table I except the SAE Photovolt data.

4. EFFECT OF SAMPLING FLOW RATE

Results from the five column sets that constituted the sampling flow-rate influence series are tabulated in Table III.

The ΔM and ΔCV of all 10 column pairs were insignificantly small, but the r value of seven of these pairs was less than the 0.990 criterion. There was one odd point in one of these seven pairs, but even after correction the r value was still significantly low. The plots of two of the other three pairs demonstrated slight correlation function translation, indicating that their r values are deceptively high.

This small but significant and reasonably consistent lack of correlation indicated that the sampling flow rate had a small, but significant, influence on smoke measurement.

There is also a consistent trend in the ΔM column of Table III. The higher flow rate (0.0085 scfs) produced lower average SN with all 10 pairs by 0.9 to 3.3.

None of the 10 ΔCV's are significantly large, so neither flow rate appeared to have intrinsic superiority.

5. EFFECT OF REFLECTOMETER CHOICE

The results of this series for the parameter values, MacBeth, Densichron, and Photovolt reflectometers, are given in Table IV. The four sets of this series consisted of 18 column pairs.

16

TABLE III. RESULTS OF SAMPLING FLOW RATE INFLUENCE INVESTIGATION

D A COLUMN IDENTIFICATION CODE	NUMBER OF POINTS PER COLUMN/ROWS INCLUSIVE FROM TABLE I	MEAN (M)	DIFFERENCE IN MEANS (ΔM)	COEFFICIENT OF VARIATION (CV)	DIFFERENCE IN COEFFICIENT OF VARIATION (ΔCV)	CORRELATION COEFFICIENT (r)	COMMENTS
SET 1							
WM8B / WM4B	19 /15-18,26-28, 34,41-44, 46-48, AND 53-56	40.2 / 42.1	1.9	40.8 / 40.8	0.0	0.986*	DISPERSION IN ALL FOUR CORRELATION PLOTS FAIRLY CONSISTENT. LOW r NOT DUE TO UNIQUE ODD POINTS.
WM8W / WM4W		47.1 / 49.2	2.1	31.3 / 32.5	1.2	0.933*	
WD8B / WD4B		40.2 / 41.9	1.7	39.0 / 40.3	1.3	0.977*	
WD8W / WD4W		44.7 / 45.6	0.9	32.6 / 35.3	2.7	0.967*	
SET 2							
WP8B / WP4B	16 /15-18,34, 41-44,47-49, AND 53-56	37.5 / 40.8	3.3	42.6 / 42.3	0.3	0.993	r IS DECEPTIVELY HIGH. PLOT DISPLAYED SLIGHT TRANSLATION.
SET 3							
MM8B / MM4B	18 /17,18,26-28, 34,41-48, AND 53-56	36.1 / 39.2	3.1	50.9 / 52.0	1.1	0.996	SLIGHT TRANSLATION OF CORRELATION LINE APPARENT ON PLOT.
MM8W / MM4W		36.9 / 39.4	2.5	50.6 / 50.8	0.2	0.993	
SET 4							
MD8B / MD4B	17 /16,18,27,28, 34,41-47,49,AND 53-56	34.5 / 36.2	1.7	51.3 / 52.2	0.9	0.989*	GOOD CONSISTENT PLOTS; UNCORRECTED RESULTS ARE PLAUSIBLE.
MD8W / MD4W		35.6 / 37.3	1.7	49.4 / 50.6	1.2	0.989*	
SET 5							
MP8B / MP4B	14 /16,18,27,28, 34,42-46, AND 53-56	32.6 / 35.8	3.2	56.4 / 52.6	3.8	0.948*	PLOT DISPLAYED ONE VERY ODD POINT. r WAS 0.988 WHEN THAT ONE WAS CORRECTED.

* SIGNIFICANT DISTINCTION IN THE UNCORRECTED DATA. SEE CRITERIA IN SECTION III PARAGRAPH 2

TABLE IV. RESULTS OF REFLECTOMETER INFLUENCE INVESTIGATION

DATA COLUMN IDENTIFICATION CODE	NUMBER OF POINTS PER COLUMN/ROWS INCLUSIVE FROM TABLE I	MEAN (M)	DIFFERENCE IN MEANS (ΔM)	COEFFICIENT OF VARIATION (CV)	DIFFERENCE IN COEFFICIENT OF VARIATION (ΔCV)	CORRELATION COEFFICIENT (r)	COMMENTS
SET 1	30						
WM8B	/16-20,	54.5	0.7	49.1	0.4	0.960*	
WD8B		33.8		48.7			
WM8B	22-25,28,	34.5	2.9	49.1	0.9	0.981*	LOW r ATTRIBUTABLE TO
WP8B	29,32-34,37-40,	31.6		48.2			FROM 1 TO 5 ODD POINTS
WD8B	42-49,AND	33.8	2.2	48.7	0.5	0.984*	IN EACH COLUMN PAIR
WP8B	52-56	37.6		48.2			OF 30 POINTS. DISPERSION
MM5B		30.3	0.6	60.6	2.6	0.990	CONSISTENT EXCEPT FOR
MD8B		30.9		59.2			THESE FEW ODD POINTS.
MM8B		30.3	0.7	60.3	2.2	0.971*	(SEE FIGURES 3 AND 4)
MP8B		29.6		63.0			
MD8B		30.7	1.3	59.2	4.8	0.988*	
MP8B		29.6		63.0			
SET 2	18						
WM3W	/17-25,29,	36.1	3.3	42.8	4.5*	0.995	
WD8W		35.8		47.3			
WM8W	32-34,	39.1	2.7	42.8	3.7	0.994	
WP8W	37-40,AND 42	36.4		46.5			
WD8W		35.8	0.6	47.3	0.8	0.999	
WP8W		36.4		46.5			
MM8W		27.1	3.4*	70.5	9.6*	0.971*	DISTINCTION DUE TO
MD8W		30.5		60.9			1 VERY ODD POINT OF
							18 POINTS TOTAL
MM8W		27.1	2.0	70.5	6.8*	0.991	
MP8W		29.1		63.7			
MD8W		30.5	1.4	60.9	2.8	0.972*	TWO VERY ODD POINTS
MP8W		29.1		63.7			APPARENT
SET 3	15						
WM4B	/15-18, 33,41-44,	42.0	1.6	44.3	3.3	0.948*	
WD4B	47, 48, AND 53-56	43.6		41.0			
WM4B		42.0	2.4	44.3	0.9	0.975*	LOW r ATTRIBUTABLE TO
WP4B		39.6		43.4			2 TO 5 ODD POINTS IN
WD4B		43.6	4.0	41.0	2.4	0.929*	ALL THREE PAIRS.
WP4B		39.6		43.4			
SET 4	16						
MM4B	/16,18,26-28,	35.2	0.4	58.5	4.3	0.980*	
MD4B	34,41-46,AND	35.6		54.2			ONE ODD POINT IN
	53-56						COLUMN MM4B WAS THE
MM4B		35.2	0.1	58.5	3.9	0.981*	CAUSE OF LOW r IN BOTH
MP4B		35.1		54.6			CASES.
MD4B		35.6	0.5	54.2	0.4	0.989	
MP4B		35.1		54.6			

* SIGNIFICANT DISTINCTION IN THE UNCORRECTED DATA. SEE CRITERIA IN SECTION III PARAGRAPH 2.

With uncorrected data, ΔM was significantly large with 1 of the 18 pairs, and ΔCV was significantly large with 3 of 18 pairs. The correlation coefficient was significantly low with 12 of 18 pairs.

The first set contained six column pairs, each with 30 points per column. Five of the six pairs displayed significantly low r, but this lack of correlation was attributable to the existence of 1 to 5 odd points per column pair. Figures 3 and 4 show two of the correlation plots in question. Correction of the odd points resulted in all correlation coefficients being greater than 0.990.

Where low correlation appeared in the other three sets, it was also attributable to from 1 to 4 odd points in each column pair. Correction resulted in r being greater than 0.990 in all cases.

There are no trends evident in the ΔM and ΔCV columns of Table IV. Choice among the three reflectometers did not appear to influence the resultant SN; no one of the three displayed superiority.

6. EFFECT OF REFLECTOMETER BACKGROUND SHADE

The results of this series with 10 column pairs arranged into three sets are shown in Table V.

The effects of reflectometer background shade and filter medium choice are closely coupled. It was generally evident that reflectometer background shade had no significant influence when used with Millipore filter medium, but had significant influence when used with Whatman.

Of the five Whatman medium pairs, four displayed significantly high ΔM, all five displayed significantly high ΔCV, and four displayed signifi----+' r. The contrary was true with the five Millipore pairs. None of the ΔM o. ΔCV was significantly large. One of the pairs displayed r less than 0.990, but this was attributable to one odd point out of 17 per column in the pair.

The magnitude of the effect with Whatman paper was markedly displayed in the plots. Two of the five plots are included here as Figures 5 and 6. All five Whatman plots displayed translated functions, indicating that the uncorrected data correlation coefficients were deceptively high.

19
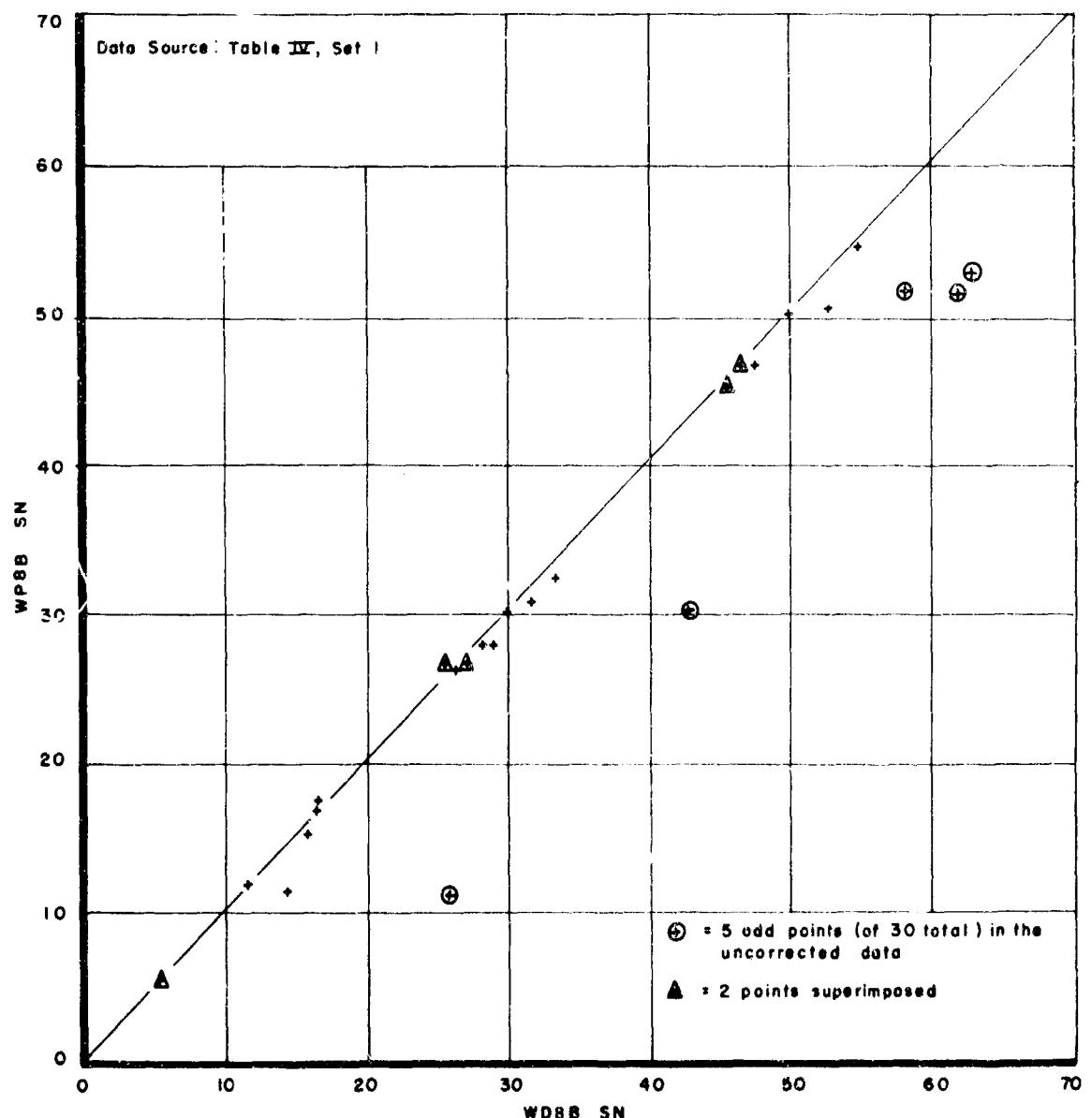
Figure 3.   Correlation Plot of WD8B-WP8B Showing Odd Points
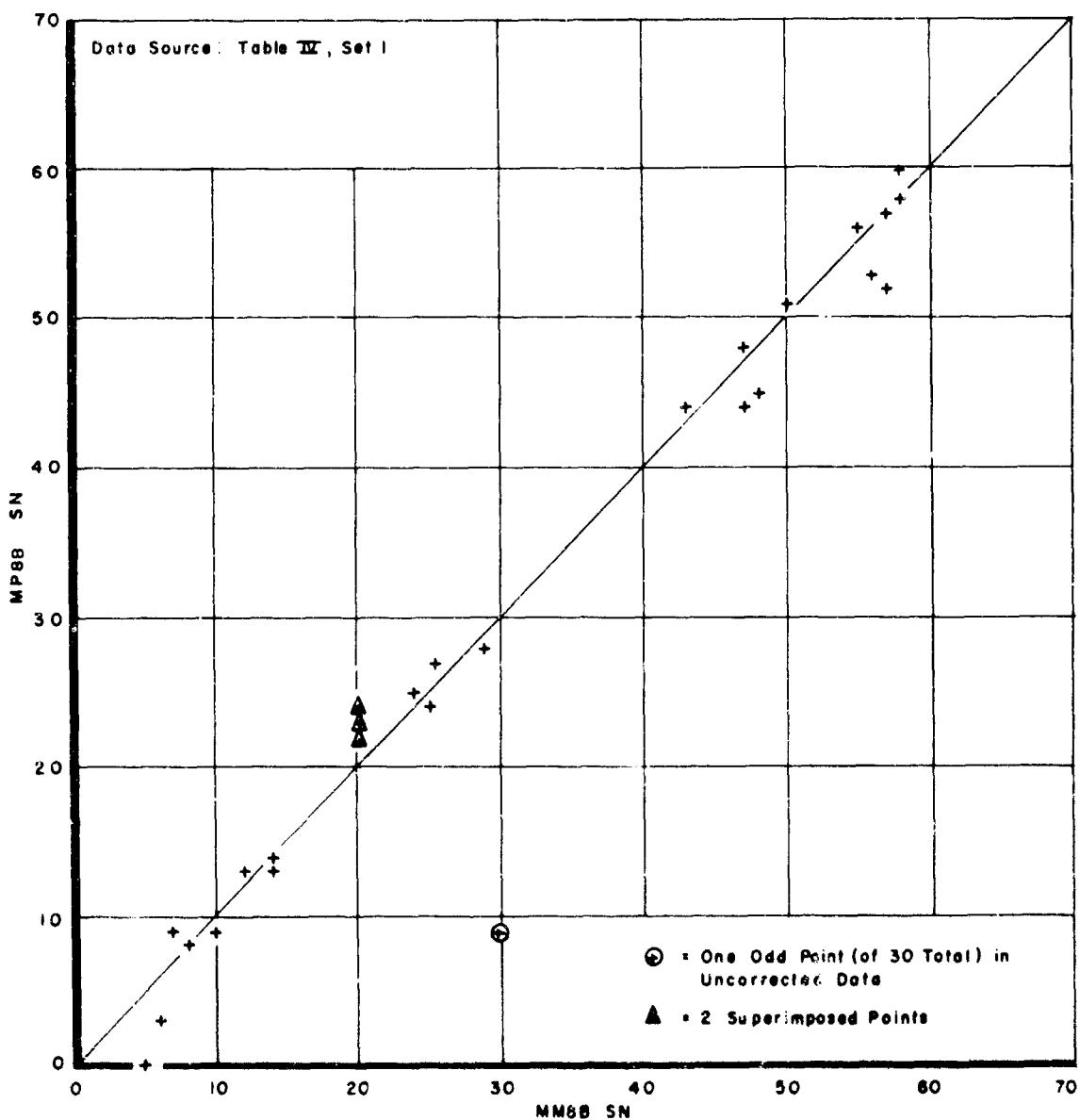
Figure 4. Correlation Plot of MM8B-MP3B Showing Odd Points

TABLE V - RESULTS OF REFLECTOMETER BACKGROUND
SHADE INFLUENCE INVESTIGATION

Data Column Identification Code	Number of Points per Column/Rows from Table I Inclusive	Mean (M)	Difference in Means (ΔM)	Coefficient of Variation (CV)	Difference in Coefficients of Variation (ΔCV)	Correlation Coefficient (r)	Comments
Set 1							
WM8B	35	36.3	7.3*	47.7	10.9*	0.982*	Both are deceptively high-correlation function strongly translated. See Figure 5
WM8W	/17-25, 27-29,	43.6		36.8			
WD8B	32-35,	35.6	5.0*	47.6	7.3*	0.985*	
WD8W	37-49, and 51-56	40.6		40.3			
MM8B		32.2	0.4	57.9	1.7	0.996	
MM8W		32.6		59.6			
MD8B		33.1	1.4	55.9	2.7	0.991	One odd point. r = 0.993 if corrected
MD8W		34.5		53.2			
Set 2							
WM4B	15	42.5	7.4*	38.5	8.5*	0.989*	r deceptively high. Plots showed both correlation lines to be very translated.
WM4W	/18,26-28,	49.9		30.0			
WD4B	34, 41-44,	42.5	3.3	37.9	4.0*	0.963*	
WD4W	46-47, and 53-56	45.8		33.9			
MM4B		39.0	0.3	50.2	0.4	0.997	
MM4W		39.3		49.8			
MD4B		38.1	1.1	49.2	1.6	0.999	
MD4W		39.2		47.6			
Set 3							
WP8B	17	29.2	6.9*	46.5	7.8*	0.995	r deceptively high. Plot showed obvious translation (Figure 6)
WP8W	/18-20, 22-25,	36.1		38.7			
MP8B	32-34,	26.0	2.5	64.8	4.3	0.986*	One odd point r > 0.990 if corrected
MP8W	37-40, and 42	27.5		60.5			

* Significant distinction in the uncorrected data. See criteria in Section III, Paragraph 2.
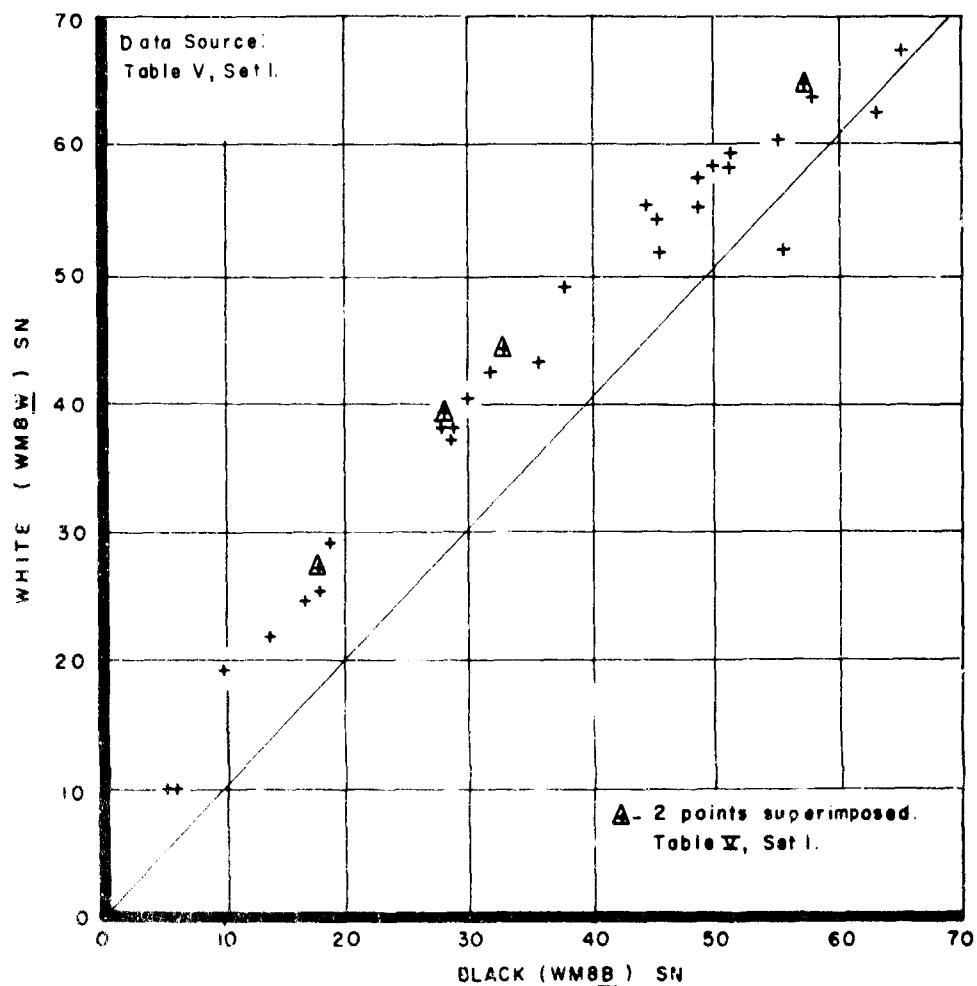
Figure 5.   Correlation Plot of WM8B-WM8W Showing Influence of Background
Shade with Whatman Medium
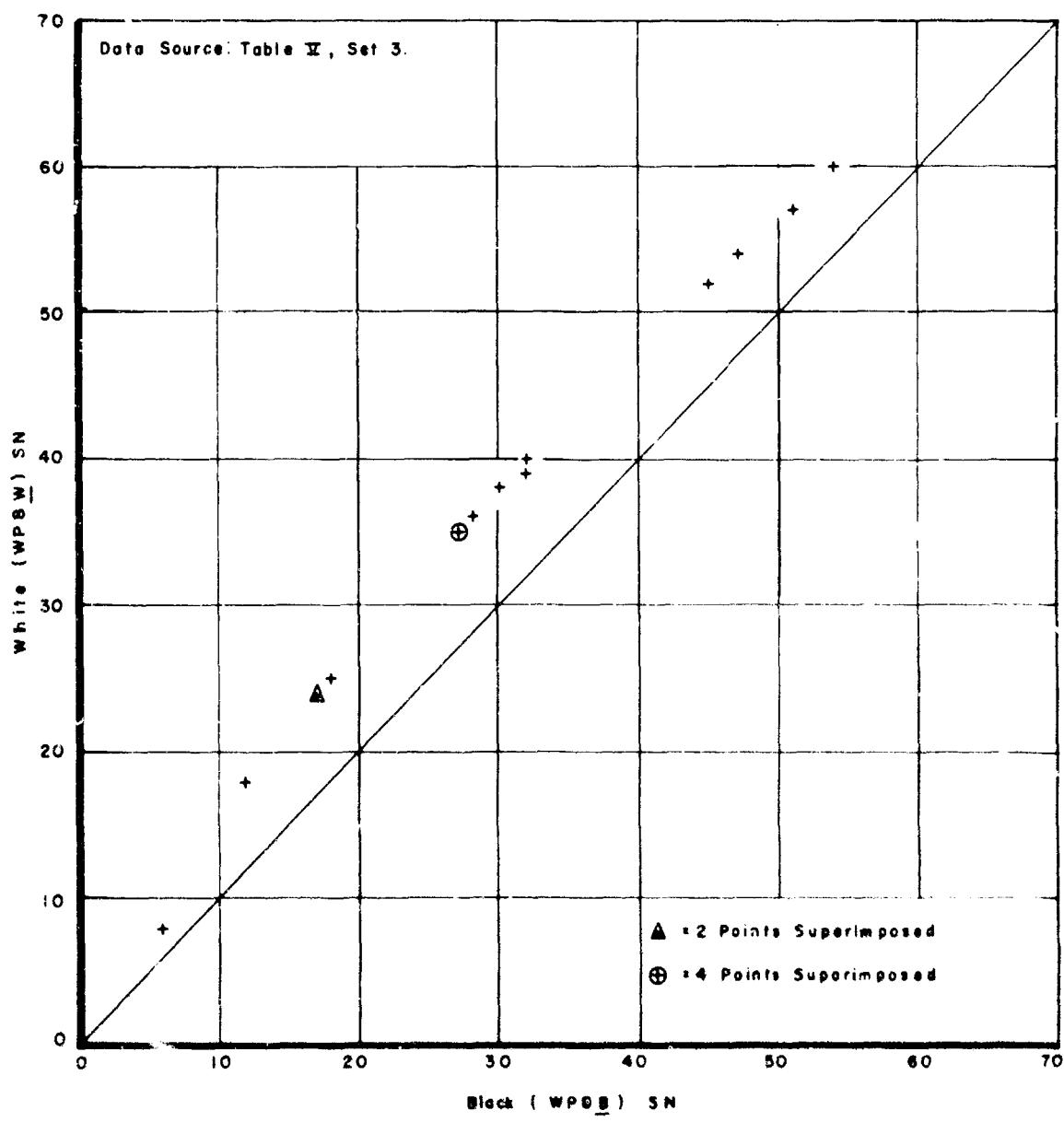
Figure 6.   Correlation Plot of WP8B-WP8W Showing Influence of Background
Shade with Whatman Medium

The white background produced higher averages (M) and lower dispersion per unit mean (CV) with Whatman medium. White appeared to be superior when used with the Whatman medium.

With the Millipore medium, background did not have significant influence on the results, so there is no superiority of one background value over the other.

7. EFFECT OF FILTER MEDIUM CHOICE

Table VI gives the results of this series with 10 column pairs arranged into four sets.

The ΔM, ΔCV, and r indicated significant difference in results taken with Millipore vs Whatman media.

The coupling of reflectometer background shade and filter medium choice effects is also very noticeable in this series. With white background, Whatman gave significantly higher averages of 6.1 to 11.0 with all five pairs. With the five black background column pairs, the Whatman mean is higher than the Millipore mean by 2.5 to 4.1, but these ΔM's appeared to be significantly large in only two of the five cases.

Not only were the differences large overall, but the correlation plots revealed that the differences in results from Whatman versus Millipore media were consistently greatest in the important region of $^\circ N$ 15 to 45. Figures 7 and 8 show two of these correlation plots.

Whatman displayed consistent superiority over Millipore medium in all 10 comparisons. The Whatman column means were highest in all 10 cases, and the amount of dispersion per unit mean (CV) is lowest for Whatman in all 10 cases. Background choice affects the magnitude of this superiority. Whatman was much more superior to Millipore on the white background. The same trend was consistently evident with black background, although the magnitude of Whatman's superiority was less than that of Millipore.

TABLE XI - RESULTS OF FILTER MEDIA
INFLUENCE INVESTIGATION

Data Column Identification Code	Number of Points per Column/Rows Inclusive from Table I	Mean (M)	Difference in Means (ΔM)	Coefficient of Variation (CV)	Difference in Coefficient of Variation (ΔCV)	Correlation Coefficient (r)	Comments
Set 1							
WM8B	35	36.3	4.1*	47.7	10.2*	0.956*	All plots showed Whatman results much higher than Millipore for SN less than about 45. Less difference but some trend for SN greater than about 45. See Figure 7.
MM8B	/17-25,27-29,	32.2		57.9			
WM8W	32 - 35,	43.6	11.0*	36.8	22.8*	0.951*	
MM8W	37 - 49,	32.6		59.6			
WD8B	and 51-56	35.6	2.5	47.6	8.3*	0.927*	
MD8B		33.1		55.9			
WD8W		40.6	6.1*	40.3	12.9*	0.959*	
MD8W		34.5		53.2			
Set 2							
WM4B	18	43.0	3.8	39.2	12.8*	0.960*	Both plots showed difference between Whatman and Millipore greatest for SN VI 45. See Figure 8.
MM4B	/17, 18,	39.2		52.0			
WM W	26-28, 34,	50.1	10.7*	31.1	19.8*	0.968*	
MM4W	41-48, and	39.4		50.9			
	53-56						
Set 3							
WD4B	16	41.8	4.0	37.9	10.2*	0.819*	Much dispersion revealed on plot.
MD4B	/16,18,	37.8		48.1			
WD4W	26-28, 34,	45.4	6.6*	33.3	13.2*	0.916*	Plot showed difference greatest at SN ≤ 45
MD4W	41-44, 46,	38.8		46.5			
	47, and						
	53 - 56						
Set 4							
WP8B	18	27.9	3.3*	51.1	20.0*	0.966*	Difference between Whatman and Millipore patently greatest for SN ≤ 45.
MP8B	/16-20,	24.6		71.1			
	22-25, 29,						
WP8W	32 - 34,	34.6	8.4*	43.7	21.7*	0.935*	
MP8W	37-4C,and	26.2		65.4			
	42						

* Significant distinction in the uncorrected data. See criteria in Section III, Paragraph 2

Figure 7. Correlation Plot of WD8W-MDSW Showing Greatest Difference Between Results with Two Filter Media at Low SN

27
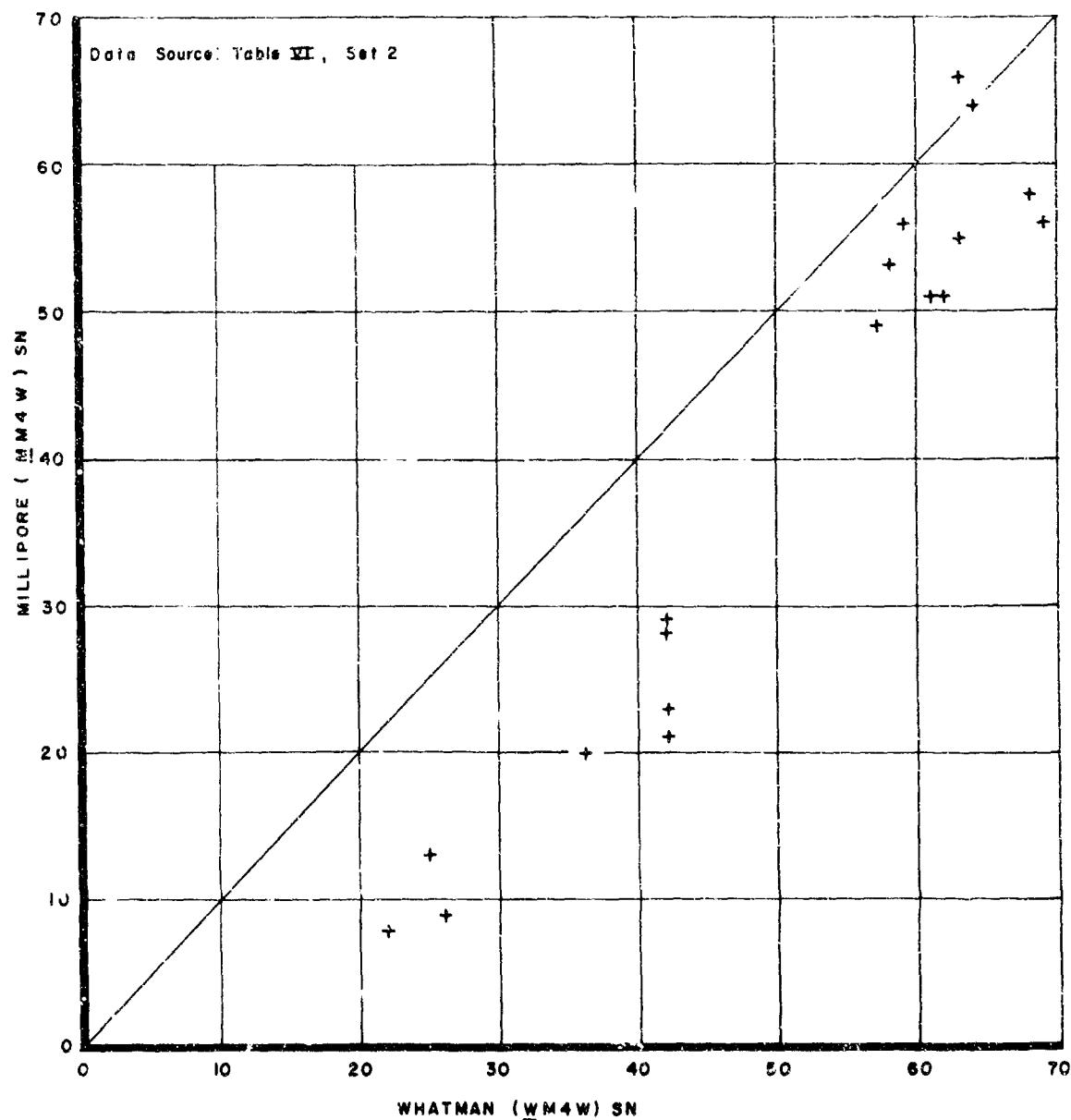
Figure 8. Correlation Plot of WM4W-MM4W Showing Greatest Difference
Between Results with Two Filter Media at Low SN

SECTION IV

DISCUSSION OF RESULTS

1. SAMPLING FLOW RATE CONSIDERATIONS

The analysis indicated that doubling the sampling flow rate (from 0.0041 to 0.0085 scfs) produced a reduction of 2% to 9% in SN.

In 1954, Watson reported that the need for isokinetic sampling (i.e., matching the sampling velocity at the probe entrance to the surrounding stream velocity) became greater as particle size increased (Reference 4). Recent work has indicated that the particulate matter in aero engine exhaust is of such small size as to make the need for isokinetic sampling superfluous (References 1, 2, 3, and 5). The results of this analysis tend to corroborate that recent work. The small sampling flow-rate effect must be considered, but it does not appear to be large enough to justify the complexity and effort involved in isokinetic sampling. Merely specifying a standard flow-rate value seems to be proper and sufficient.

Some smoke measuring systems employ sample volume and sampling time measurements to determine flow rate. The analysis also indicated that such a more precise yet laborious procedure for determining flow rate is superfluous. The analysis tends to indicate that variations in flow rate of as much as 10% will produce variation in results (SN) of less than 1%.

2. CHOICE OF REFLECTOMETER

The analysis indicated that all three reflectometers produced substantially the same results. The differences in data column means were 2% to 11%, but there was no evidence of any one reflectometer producing superior quality results.

3. CHOICE OF FILTER MEDIUM AND REFLECTOMETER BACKGROUND SHADE

The results showed that filter media and background shade effects were coupled.

Combinations of filter media and reflectometer background shades are ranked in Table VII. The M and CV averages in Table VII were prepared from Tables V and VI. The corresponding SD averages were prepared from computer calculated SD not reproduced in this report.

TABLE VII. SUPERIORITY RANKING OF FILTER
MEDIUM-REFLECTOMETER BACKGROUND SHADE COMBINATIONS

Combination	Average Mean (Maximize for Superiority)	Average CV (Minimize for Superiority)	Average Standard Deviation	Ranking
Whatman/white	43.0	37.1	15.4	Best Combination
Whatman/black	36.9	44.6	16.1	
Millipore/white	34.3	56.4	18.3	Either Combination Least Desirable (Insignificant Difference Between These Two)
Millipore/black	33.4	55.9	18.1	

Whatman with white background displays significant superiority (highest M, lowest CV) over all other combinations.

With Millipore, the differences between results on either white or black background are not significant.

The combined results given in Table VII are for the full range of smoke levels investigated during the tests (approximately SN of 5 to 70). However, the correlation plots revealed that the distinction between Whatman and Millipore media is even greater in the most important region of SN from about 10 to 45 (see Figures 7 and 8).

The effect of black background is to decrease the magnitude of Whatman superiority by decreasing the overall average SN from 43.0 on white to 36.9 on black (Table VII). It is significant to note that the higher CV of Whatman on black versus Whatman on white is largely attributable to this reduction in SN; the amount of dispersion in Whatman data is about the same with both black and white background shades. Conversely, the even higher CV averages of both

30

Millipore combinations are primarily attributable to more dispersion (higher standard deviations), in addition to lower column means.

The superiority of Whatman paper has been previously implied if not explicitly denoted. Bagnetto (Reference 2) evaluated three smoke measuring systems and concluded that the Von Brand system, which uses Whatman No. 4 medium, was significantly superior to the AED system that used Millipore. (The third system, the B. P. Hartridge nonfiltration type based on light absorption, was ranked slightly above the AED system, yet still significantly below the filtration type system using Whatman medium.)

It may be possible to reconcile the differences in results obtained on Millipore versus Whatman media. One theoretically possible tack for making the results of Millipore medium approximately equal to those of Whatman medium is discussed in Appendix V.

SECTION V

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

The subject data can be used to draw statistically valid conclusions about four parameters: sampling flow rate, choice of reflectometer, choice of filter medium, and reflectometer background shade.

Sampling flow rate had a small, yet consistent and significant, influence. The higher flow rate tested (0.0085 scfs) produced SN results that were 2% to 9% lower than results with the lower flow rate (0.0041 scfs). Since there was not enough data to compare results at each of the four engine power levels used during testing, no firm statement can be made about the need for isokinetic sampling. However, the analysis does tend to corroborate previous work that concluded that the isokinetic sampling requirement is superfluous when sampling exhaust smoke from aircraft gas turbine engines.

All three reflectometers used to rate spots (MacBeth, Densichron, and Photovolt) were found to produce essentially the same results. No one of the three demonstrated superiority.

The effects of filter media choice and reflectometer background shade are closely coupled. Whatman filter medium evaluated on the white background produced the best results. Whatman filter medium on black background gave significantly lower SN, but the dispersion of data with this combination was not significantly different from Whatman on white. Whatman on black was the second best combination.

Background shade did not significantly influence results obtained with Millipore filter medium. Results of Millipore with either background were significantly more dispersed than the results of Whatman with either background.

Whatman medium with either background was superior to Millipore with either background.

2. RECOMMENDATIONS

The SAE system for measuring aero engine exhaust smoke should specify a single sampling flow-rate value. It should also specify a rotameter or other simple device for direct measurement of flow rate. The influence of sampling flow rate on results is not great enough to justify the need for isokinetic sampling, nor is the influence great enough to justify more precise, indirect, means of determining sampling flow rate.

The SAE document should either specify use of any of the three reflectometers tested, or otherwise ensure that an inferior instrument is not allowed.

The use of Whatman No. 4 filter paper and black reflectometer background (i.e., absolute background reflectance of 5% or less) should be specified. The analysis determined that white background was superior, but its use is undesirable. There is some evidence that white background tends to exaggerate the differences between values of all parameters, as the analysis indicates it does with filter media. In the first set of Table VI, the ΔCV of columns WM8$\underline{B}$ and WD8$\underline{B}$ is virtually identical: 47.7 - 47.6 = 0.1. Yet the ΔCV of WM8$\underline{W}$ and WD8$\underline{W}$ is much larger: 40.3 - 36.8 = 3.5. Since it is impossible to specify limitations on all parameters, the use of white background could cause poorer results by affecting unspecified parameters. The analysis indicates that use of black background does not appreciably increase data dispersion, although its use will cause a slight loss in precision due to the absolute value of SN being lower. This, in the author's opinion, is a justifiable tradeoff to guard against unknown factors.

APPENDIX I

SUMMARY OF TEST PROGRAM AND MEASUREMENT SYSTEM EVALUATED

By June of 1969, SAE Committee E-31 had decided that the standard measurement system should be an indirect, filtration type not unlike most systems in use today. A test program was conducted by the Committee at the Federal Aviation Administration's experimental center in Atlantic City, New Jersey. The program was intended to experimentally examine the tentative smoke measurement system.

A J-57 turbojet engine was used to generate smoke. The elements of the measurement system being evaluated were provided by various Committee members.

The design of the experimental program is unknown to this author, although it is known that the testing sequence used did approach being random. Data was obtained with different combinations of values of the following:

- engine power level (4 values)
- filter media (2 types)
- filter media holder (2 types)
- sample size (standard volume of exhaust gas; 4 values for each of the 2 types of filter media)
- sampling flow rate (2 values)
- sampling probe angular orientation with respect to the direction of engine exhaust gas flow (3 values)
- sampling probe position along the engine exhaust gas stream flow path (2 values)
- sampling line length (2 values)
- sampling line size (diameter - 2 values)
- sampling line material (2 types)
- sampling line temperature (2 values)

Data reduction introduced two more parameters with variable values:

- reflectometer (3 types)
- reflectometer background shade (2 values)

Not all parameter values were changed for all runs. Yet there was enough change so that no two rows of Table I contained all the same parameter values.

More than 200 data points were taken.

The basic configuration of the measurement system and operating procedure were the same throughout testing. A given sample size was drawn at a given flow rate from the engine exhaust through the sampling probe, sampling line, and filter media holder with a vacuum pump. A rotameter, positive displacement volume space meter, and pressure and temperature gauges were used downstream of the pump to measure the sample before it was discharged to the atmosphere. The flow time of each sample was also measured. The system was heated throughout testing. A filter holder bypass line was used to maintain flow rate in the system when a sample was not being taken.

APPENDIX II

DATA REDUCTION PROCEDURES

1. SAE GROUP

The smoke spots were read with meter-background combinations to rate
them in terms of either absolute reflectance or optical density, depending on
which meter was being used. These readings were then used to calculate SN.
The definition of SN, the dimensionless term used to quantify smoke emission,
is:

$$SN = 100 \left(1 - \frac{R_s}{R_w}\right), \text{ where} \tag{1}$$

Rs - absolute reflectance of the sample spot

Rw - absolute reflectance of clean filter media

The relationship between optical density (OD) and absolute reflectance (R)
is:

$$OD = \log_{10}\left(\frac{100}{R}\right) \tag{2}$$

The SAE data reduction group used graphs combining Equations 1 and 2 to
obtain SN for spots used in terms of optical density (the MacBeth meter).
Equation 1 was used to calculate SN for spots read in terms of absolute re-
flectance (the Photovolt meter).

SN will vary with the sample size. It has long been accepted to report SN
and other quantifiers of smoke for a certain sample size (standard cubic feet)
per unit filter medium area (square inch). This quantity is termed "Q."

It has also been accepted practice to use a specific Q value dependent on
filter medium choice. The "standard" Q for Whatman medium is 0.300 scf/sq
in, and the value used for Millipore is 0.0565 scf/sq in. These are widely
used, although apparently arbitrary, values.

The SAE Group calculated Q values, and then plotted these as abscissa
versus the corresponding SN as ordinate on log-log paper. A curve was then
fitted to these points, and then the SN values were read-off for Q values of

0.300 and 0.0565 scf/sq in for Whatn and Millipore filter media, respectively. These SN are the values reported in Table I.

2. GE GROUP

The GE Group's procedure differed from that of the SAE Group in the manner in which the effect of different sample sizes was weighed. The GE team has a standard smoke-spot data-reduction routine based on the use of "loading curves" for Whatman and Millipore media. The loading curve is a plot of micrograms of carbon (smoke particulate matter) as abscissa versus optical density of the resultant spot as ordinate (Figure 9). Spot size is a necessary parameter of such curves.

The GE data reduction routine was completely computerized. All raw data to compute SN and Q were input (reflectometer readings, sample size, etc.). The routine calculated SN and corresponding Q, and then based on leading curve factors, "corrected" each SN to the "proper" value for Q of 0.300 or 0.0565 scf/sq in, depending on which filter media was used. The resultant SN's from this procedure are those listed in Table I.
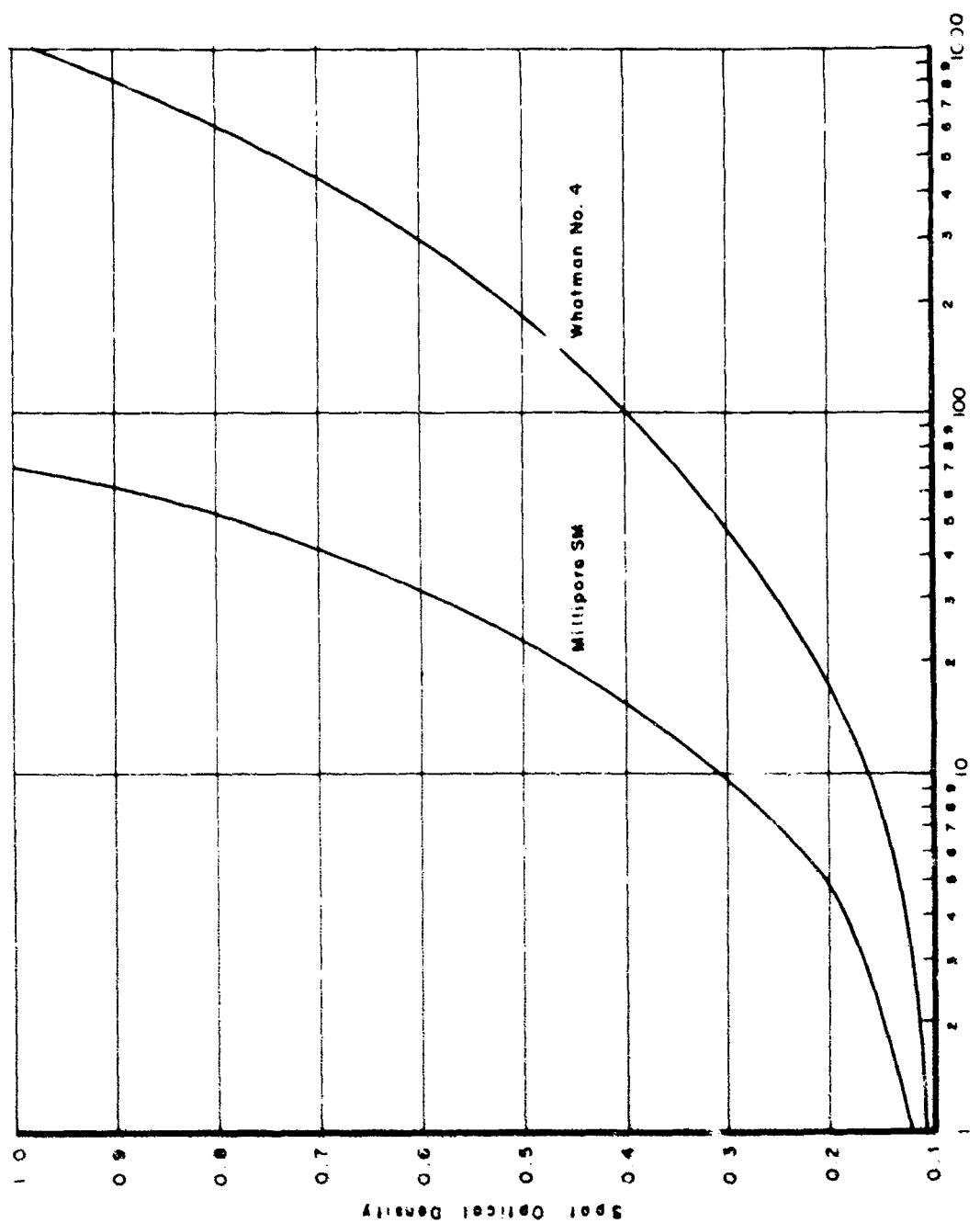
Figure 9. Loading Curves for Whatman and Millipore Filter Media

APPENDIX III

ANALYSIS COMPUTER PROGRAM CORRE1

The CORRE1 calculation routine is written in FORTRAN IV, Version 13
(Reference 10). The short main routine "MP" calls the primary subroutine
"MAIN." MAIN in turn calls the subroutines "CORRE" and "GP." The final
subroutine "DATA" is a short dummy element used only to satisfy a call from
CORRE. (DATA is included to avoid having to modify CORRE.) Subroutines
CORRE and GP were taken from References 7 and 8, respectively.

MAIN ROUTINE MP

```
C EVALUATION OF COMM. E-31 TEST DATA  -  ROUTINE  CORRE1
C
C                  INPUT  -  MT       -  NUMBER OF COLUMNS OF DATA (12 MAX.
C                                        PER CASE).
C                         -  NT       -  ROWS OF DATA PER COLUMN.  50 MAX , AND
C                                        MUST NOT BE LESS THAN MT.
C                         -  NC       -  NUMBER OF CASES. EACH SET OF 12 OR FEWER
C                                        COLUMNS,OR EACH DIFFERENT NT VALUE,
C                                        CONSTITUTE A CASE.
C                         -  JP       -  NUMBER OF COLUMN PAIRS TO BE CORRELATED.
C                         -  SNPC(J)  -  ALPHAMERIC COLUMN CODE (6 SPACES MAX. ).
C                                        J IS THE COLUMN INDEX NUMBER.
C                         -  JX(IJ),J2(IJ)
C                                     -  INDICES OF COLUMNS TO BE CORRELATED.
C                                        IJ = 1 IS THE FIRST PAIR, ETC.
C                         -  SN(I,J)  -  DATA, INPUT COLUMN BY COLUMN.
C                         -  RI(I)    -  A 42 SPACE MESSAGE OF WHICH DATA ROWS
C                                        WERE INPUT. I IS A DUMMY SUBSCRIPT.
C                  OUTPUT -  DATA, COLUMN BY COLUMN, WITH COLUMN IDENTIFYING
C                            CODE AND STATEMENT OF WHICH ROWS WERE INPUT.
C                         -  MEANS,SD,STANDARDIZED MEANS AND SD, AND COEFFICIENTS
C                            OF VARIATION, FOR ALL INPUT COLUMNS.
C                         -  CORRELATION COEFFICIENTS FOR SELECTED COLUMN
C                            PAIRS.
C                         -  PLOTTING OF EACH CORRELATED
C                            COLUMN PAIR.
C
      DIMENSION Y(100)
      READ(5,10) NC
      IC = 1
   50 READ(5,10) NT
      CALL MAIN (Y,NT)
      IC = IC+1
      IF(IC-NC) 50,50,60
   10 FORMAT(I5)
   60 STOP
      END
```

PRIMARY SUBROUTINE MAIN

```
      SUBROUTINE MAIN (Y,NT)
      DIMENSION XBAR(12),STD(12),RX(144),R(78),B(12),D(12),T(12),AMN(12)
     1,SD(12),SAM(12),SSD(12),JX(20),JZ(20),SNPC(12),CV(12)
     2SN(50,12),X(600),Y(NT,2),A(2),PLOT(36,60),RP(20),RI(7)
C
      READ(5,10)     NT,JP,(SNPC(J),J=1,NT)
      READ(5,11) (JX(IJ),JZ(IJ), IJ=1,JP)
      READ(5,12) ((SN(I,J),I=1,NT),J=1,NT)
      READ(5,22) (RI(I), I=1,7)
C
C     COMPUTE MEANS, STANDARD DEVIATIONS, AND COEFFICIENTS OF VARIATION
.
      IU=1
      M=NT
      N=NT
      JS=0
      DO 60 J=1,M
      DO 50 I=1,N
      JS=JS+1
   50 X(JS) = SN(I,J)
   60 CONTINUE
      CALL CORRE (N,M,IU,X,XBAR,STD,RX,R,B,D,T)
      DO 70 J=1,N
      CV(J) = 100.*STD(J)/XBAR(J)
      AMN(J) = XBAR(J)
   70 SD(J) = STD(J)
C
C     COMPUTE AVERAGE MEANS AND AVERAGE STANDARD DEVIATIONS, AND OUTPUT
C     WITH INPUT DATA AND MEANS, STANDARD DEVIATIONS AND COEFF. OF VARIATION
C
      N = NT
      M = 2
      IS=0
      DO 80 I=1,N
      IS=IS+1
   80 X(IS) = XBAR(I)
      DO 85 I=1,N
      IS=IS+1
   85 X(IS) =STD(I)
      CALL CORRE (N,M,IU,X,XBAR,STD,RX,R,B,D,T)
      AAMN = XBAR(1)
      ASD = XBAR(2)
      N = NT
      DO 90 J=1,NT
      SAM(J) = AMN(J)/AAMN
   90 SSD(J) = SD(J)/ASD
      WRITE(6,19) (SNPC(J),J=1,NT)
      DO 95 I=1,NT
   95 WRITE(6,18) (SN(I,J),J=1,NT)
      WRITE(6,23) (RI(I), I=1,7)
      WRITE(6,13)
      N =NT
      DO 100 J=1,NT
  100 WRITE(6,14) SNPC(J),N,AMN(J),SD(J),SAM(J),SSD(J),CV(J)
      WRITE(6,15) AAMN,ASD
```

```
C
C      COMPUTE AND OUTPUT CORRELATION COEFFICIENTS
C
       WRITE(6,16)
       M =2
       IJ=1
   110 IS = 0
       J1 = JX(IJ)
       J2=JZ(IJ)
       DO 120 I=1,N
       IS=IS+1
   120 X(IS) = SN(I,J1)
       DO 125 I=1,N
       IS=IS+1
   125 X(IS) = SN(I,J2)
       CALL CORRE (N,M,IO,X,XBAR,STD,RX,R,B,D,T)
       WRITE(6,17) SNPC(J1),SNPC(J2),R(2)
       RP(IJ) = R(2)
       IJ=IJ+1
       IF(IJ-JP) 110,110,130
C
C      X-Y PLOTTING OF CORRELATED COLUMNS
C
   130 CONTINUE
       INTEGER S,W
       DATA A/1H.,1H+/
       N=2
       M = NT
       L=6
       S=30
       W=60
       LN=36
       IJ=1
   150 WRITE(6,21)
       J1 = JX(IJ)
       J2 = JZ(IJ)
       DO 160 I=1,M
       X(I) = SN(I,J1)
       Y(I,1) = SN(I,J1)
   160 Y(I,2) = SN(I,J2)
       CALL GP(X,Y,L,S,M,N,W,LN,A,PLOT)
       WRITE(6,20) SNPC(J1),SNPC(J2),RP(IJ)
       IJ = IJ+1
       IF(IJ-JP) 150,150,200
C
C
    10 FORMAT(2I5,/8(4X,A6))
    11 FORMAT(4(I5,I5,10X))
    12 FORMAT(16F5.1)
    13 FORMAT(1H1,30X,60H AVERAGES, STANDARD DEVIATIONS AND COEFFICIENTS
      1OF VARIATION///15X,7H COLUMN,
      2  7X,7H NUMBER,3X,11H ARITHMETIC,5X,9H STANDARD,5X,13H STANDARDIZE
      3D,6X,13H STANDARDIZED,4X,16H COEFFICIENTS OF/10X,
      4  16H (PARAMETER SET),2X,8H OF DATA,6X,5H MEAN,8X,10H DEVIATION,
      5  5X,11H ARITH MEAN,5X,15H STAN DEVIATION,4X,15H VARIATION (CV)/26X
      6,12H (SN) POINTS,3X,6H (AMN),10X,5H (SD),8X,11H (AMN/AAMN),7X,9H (
```

```
       7SD/ASD)9X,15H (100 X SD/AMN)//)
    14 FORMAT(16X,A6,8X,I2,5F16.3)
    15 FORMAT(1H0,40X,34H AVERAGE ARITHMETIC MEAN (AAMN)   =,F10.3/40X,36H
      1 AVERAGE STANDARD DEVIATION (ASD)   =,F10.3///////)
    16 FORMAT(45X,2H X,10X,24H CORRELATION COEFFICIENT/45X,2H Y,14X,16H B
      1ETWEEN X AND Y//)
    17 FORMAT(42X,A6/42X,A6,10X,F10.5,//)
    18 FORMAT(10X,12F10.2)
    19 FORMAT (1H1,60X,11H INPUT DATA//10X,12(4X,A6)//)
    20 FORMAT (1H0,50X,18H ABSCISSA (X) IS   ,A6/51X,18H ORDINATE (Y) IS
      1,A6/40X,43H CORRELATION COEFFICIENT BETWEEN X AND Y  =,F10.5)
    21 FORMAT (1H1,40X,35H PLOTTING OF CORRELATED COLUMN PAIR//30X,44H PE
      1RFECT CORRELATION LINE IS DRAWN WITH DOTS/30X,40H ACTUAL COLUMNS P
      2LOTTING IS WITH CROSSES)
    22 FORMAT (7A6)
    23 FORMAT (1H0, 3X,84H THIS DATA IS FROM THE FOLLOWING ROWS,INCLUSIVE
      1,READING FROM COLUMN TOP TO BOTTOM - ,7A6)
   200 RETURN
       END
```

SUBROUTINE CORRE

```
C                                                                    CORRE001
C      ..........................................................CORRE002
C                                                                    CORRE003
C         SUBROUTINE CORRE                                           CORRE004
C                                                                    CORRE005
C         PURPOSE                                                    CORRE006
C            COMPUTE MEANS, STANDARD DEVIATIONS, SUMS OF CROSS-PRODUCTS CORRE007
C            OF DEVIATIONS, AND CORRELATION COEFFICIENTS.            CORRE008
C                                                                    CORRE009
C         USAGE                                                      CORRE010
C            CALL CORRE (N,M,IO,X,XBAR,STD,RX,R,B,D,T)               CORRE011
C                                                                    CORRE012
C         DESCRIPTION OF PARAMETERS                                  CORRE013
C            N      - NUMBER OF OBSERVATIONS.                        CORRE014
C            M      - NUMBER OF VARIABLES.                           CORRE015
C            IO     - OPTION CODE FOR INPUT DATA                     CORRE016
C                     0 IF DATA ARE TO BE READ IN FROM INPUT DEVICE IN THECORRE017
C                         SPECIAL SUBROUTINE NAMED DATA.  (SEE SUBROUTINES CORRE018
C                         USED BY THIS SUBROUTINE BELOW.)            CORRE019
C                     1 IF ALL DATA ARE ALREADY IN CORE.             CORRE020
C            X      - IF IO=0, THE VALUE OF X IS 0.0.                CORRE021
C                     IF IO=1, X IS THE INPUT MATRIX (N BY M) CONTAINING CORRE022
C                         DATA.                                      CORRE023
C            XBAR   - OUTPUT VECTOR OF LENGTH M CONTAINING MEANS.    CORRE024
C            STD    - OUTPUT VECTOR OF LENGTH M CONTAINING STANDARD  CORRE025
C                     DEVIATIONS.                                    CORRE026
C            RX     - OUTPUT MATRIX (M X M) CONTAINING SUMS OF CROSS- CORRE027
C                     PRODUCTS OF DEVIATIONS FROM MEANS.             CORRE028
C            R      - OUTPUT MATRIX (ONLY UPPER TRIANGULAR PORTION OF THE CORRE029
C                     SYMMETRIC MATRIX OF M BY M) CONTAINING CORRELATION CORRE030
C                     COEFFICIENTS.  (STORAGE MODE OF 1)             CORRE031
C            B      - OUTPUT VECTOR OF LENGTH M CONTAINING THE DIAGONAL CORRE032
C                     OF THE MATRIX OF SUMS OF CROSS-PRODUCTS OF     CORRE033
C                     DEVIATIONS FROM MEANS.                         CORRE034
C            D      - WORKING VECTOR OF LENGTH M.                    CORRE035
C            T      - WORKING VECTOR OF LENGTH M.                    CORRE036
C                                                                    CORRE037
C         REMARKS                                                    CORRE038
C            N MUST BE GREATER THAN OR EQUAL TO M.                   CORRE039
C                                                                    CORRE040
C         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED              CORRE041
C            DATA(M,D) - THIS SUBROUTINE MUST BE PROVIDED BY THE USER. CORRE042
C                        (1) IF IO=0, THIS SUBROUTINE IS EXPECTED TO CORRE043
C                            FURNISH AN OBSERVATION IN VECTOR D FROM AN CORRE044
C                            EXTERNAL INPUT DEVICE.                  CORRE045
C                        (2) IF IO=1, THIS SUBROUTINE IS NOT USED BY CORRE046
C                            CORRE BUT MUST EXIST IN JOB DECK. IF USER CORRE047
C                            HAS NOT SUPPLIED A SUBROUTINE NAMED DATA, CORRE048
C                            THE FOLLOWING IS SUGGESTED.             CORRE049
C                                SUBROUTINE DATA                     CORRE050
C                                RETURN                              CORRE051
C                                END                                 CORRE052
C                                                                    CORRE053
C         METHOD                                                     CORRE054
C            PRODUCT-MOMENT CORRELATION COEFFICIENTS ARE COMPUTED.   CORRE055
```

```
C                                                                        CORRE056
C      ..........................................................•CORRE057
C                                                                        CORRE058
       SUBROUTINE CORRE (N,M,IO,X,XBAR,STD,RX,R,B,D,T)                   CORRE059
       DIMENSION X(1),XBAR(1),STD(1),RX(1),R(1),B(1),D(1),T(1)           CORRE060
C                                                                        CORRE061
C      ..........................................................•CORRE062
C                                                                        CORRE063
C         IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  CORRE064
C         C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      CORRE065
C         STATEMENT WHICH FOLLOWS.                                       CORRE066
C                                                                        CORRE067
C      DOUBLE PRECISION XBAR,STD,RX,R,B,T                                CORRE068
C                                                                        CORRE069
C         THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS    CORRE070
C         APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS      CORRE071
C         ROUTINE.                                                       CORRE072
C                                                                        CORRE073
C         THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO      CORRE074
C         CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT AND ABS IN   CORRE075
C         STATEMENT 220 MUST BE CHANGED TO DSQRT AND DABS.               CORRE076
C                                                                        CORRE077
C      ..........................................................•CORRE078
C                                                                        CORRE079
C         INITIALIZATION                                                 CORRE080
C                                                                        CORRE081
       DO 100 J=1,M                                                      CORRE082
       B(J)=0.0                                                          CORRE083
  100 T(J)=0.0                                                           CORRE084
       K=(M**+M)/2                                                       CORRE085
       DO 102 I=1,K                                                      CORRE086
  102 R(I)=0.0                                                           CORRE087
       FN=N                                                              CORRE088
       L=0                                                              CORRE089
C                                                                        CORRE090
       IF(IO) 105, 127, 105                                              CORRE091
C                                                                        CORRE092
C      DATA ARE ALREADY IN CORE                                          CORRE093
C                                                                        CORRE094
  105 DO 108 J=1,M                                                       CORRE095
       DO 107 I=1,N                                                      CORRE096
       L=L+1                                                             CORRE097
  107 T(J)=T(J)+X(L)                                                     CORRE098
       XBAR(J)=T(J)                                                      CORRE099
  108 T(J)=T(J)/FN                                                       CORRE100
C                                                                        CORRE101
       DO 115 I=1,N                                                      CORRE102
       JK=0                                                              CORRE103
       L=I-N                                                             CORRE104
       DO 110 J=1,M                                                      CORRE105
       L=L+N                                                             CORRE106
       D(J)=X(L)-T(J)                                                    CORRE107
  110 B(J)=B(J)+D(J)                                                     CORRE108
       DO 115 J=1,M                                                      CORRE109
       DO 115 K=1,J                                                      CORRE110
       JK=JK+1                                                           CORRE111
```

```
     115 R(JK)=R(JK)+D(J)*D(K)                              CORRF112
         GO TO 205                                          CORRF113
C                                                           CORRF114
C        READ OBSERVATIONS AND CALCULATE TEMPORARY          CORRF115
C        MEANS FROM THESE DATA IN T(J)                      CORRE116
C                                                           CORRF117
     127 IF(N-M) 130, 130, 135                              CORRE118
     130 KK=N                                               CORRE119
         GO TO 137                                          CORRE120
     135 KK=M                                               CORRF121
     137 DO 140 I=1,KK                                      CORRE122
         CALL DATA (M,D)                                    CORRF123
         DO 140 J=1,M                                       CORRE124
         T(J)=T(J)+D(J)                                     CORRF125
         L=L+1                                              CORRE126
     140 RX(L)=D(J)                                         CORRF127
         FKK=KK                                             CORRE128
         DO 150 J=1,M                                       CORRF129
         XBAR(J)=T(J)                                       CORRE130
     150 T(J)=T(J)/FKK                                      CORRF131
C                                                           CORRE132
C        CALCULATE SUMS OF CROSS-PRODUCTS OF DEVIATIONS     CORRF133
C        FROM TEMPORARY MEANS FOR M OBSERVATIONS            CORRF134
C                                                           CORRF135
         L=0                                                CORRF136
         DO 180 I=1,M                                       CORRF137
         JK=0                                               CORRF138
         DO 170 J=1,N                                       CORRF139
         L=L+1                                              CORRF140
     170 D(J)=RX(L)-T(J)                                    CORRF141
         DO 180 J=1,M                                       CORRF142
         B(J)=B(J)+D(J)                                     CORRF143
         DO 180 K=1,J                                       CORRF144
         JK=JK+1                                            CORRF145
     180 R(JK)=R(JK)+D(J)*D(K)                              CORRF146
C                                                           CORRF147
         IF(M-KK) 205, 205, 185                             CORRF148
C                                                           CORRF149
C        READ THE REST OF OBSERVATIONS ONE AT A TIME, SUM   CORRF150
C        THE OBSERVATION, AND CALCULATE SUMS OF CROSS-      CORRF151
C        PRODUCTS OF DEVIATIONS FROM TEMPORARY MEANS        CORRF152
C                                                           CORRF153
     185 KK=M-KK                                            CORRF154
         DO 200 I=1,KK                                      CORRF155
         JK=0                                               CORRF156
         CALL DATA (M,D)                                    CORRF157
         DO 190 J=1,M                                       CORRF158
         XBAR(J)=XBAR(J)+D(J)                               CORRF159
         D(J)=D(J)-T(J)                                     CORRF160
     190 B(J)=B(J)+D(J)                                     CORRF161
         DO 200 J=1,M                                       CORRF162
         DO 200 K=1,J                                       CORRF163
         JK=JK+1                                            CORRF164
     200 R(JK)=R(JK)+D(J)*D(K)                              CORRF165
C                                                           CORRF166
C        CALCULATE MEANS                                    CORRF167
```

```
C                                                                  CORRE168
   205 JK=0                                                        CORRE169
       DO 210 J=1,M                                                CORRE170
       XBAR(J)=XBAR(J)/FN                                          CORRE171
C                                                                  CORRE172
C      ADJUST SUMS OF CROSS-PRODUCTS OF DEVIATIONS                 CORRE173
C      FROM TEMPORARY MEANS                                        CORRE174
C                                                                  CORRE175
       DO 210 K=1,J                                                CORRE176
       JK=JK+1                                                     CORRE177
   210 R(JK)=R(JK)-B(J)*B(K)/FN                                    CORRE178
C                                                                  CORRE179
C      CALCULATE CORRELATION COEFFICIENTS                         CORRE180
C                                                                  CORRE181
       JK=0                                                        CORRE182
       DO 220 J=1,M                                                CORRE183
       JK=JK+J                                                     CORRE184
   220 STD(J)= SQRT( ABS(R(JK)))                                  CORRE185
       DO 230 J=1,M                                                CORRE186
       DO 230 K=J,M                                                CORRE187
       JK=J+(K*K-K)/2                                              CORRE188
       L=M*(J-1)+K                                                 CORRE189
       RX(L)=R(JK)                                                 CORRE190
       L=M*(K-1)+J                                                 CORRE191
       RX(L)=R(JK)                                                 CORRE192
   230 R(JK)=R(JK)/(STD(J)*STD(K))                                CORRE193
C                                                                  CORRE194
C      CALCULATE STANDARD DEVIATIONS                              CORRE195
C                                                                  CORRE196
       FN=SQRT(FN-1.0)                                             CORRE197
       DO 240 J=1,M                                                CORRE198
   240 STD(J)=STD(J)/FN                                           CORRE199
C                                                                  CORRE200
C      COPY THE DIAGONAL OF THE MATRIX OF SUMS OF CROSS-PRODUCTS OF CORRE201
C      DEVIATIONS FROM MEANS.                                      CORRE202
C                                                                  CORRE203
       L=-M                                                        CORRE204
       DO 250 I=1,M                                                CORRE205
       L=L+M+1                                                     CORRE206
   250 B(I)=RX(L)                                                  CORRE207
       RETURN                                                      CORRE208
       END                                                         CORRE209
```

SUBROUTINE GP

```
      SUBROUTINE GP (X, Y, L, S, M, N, W, LN, A, PLOT)              DSPLY001
C**                                                                  DSPLY002
C**    CONTROL                                                       DSPLY003
C**                                                                  DSPLY004
C                                                                    DSPLY005
C**    CALL GP (X, Y, L, S, M, W, LN, A, PLOT)                       DSPLY006
C**                                                                  DSPLY007
C**      WHERE                                                       DSPLY008
C**          X = ARRAY OF INDEPENDENT VALUES, DIMENSIONED X(M).      DSPLY009
C**          Y = ARRAY OF SETS OF DEPENDENT VALUES, DIMENSIONED Y(M,N). DSPLY010
C**          L = NUMBER OF LINES TO BE SKIPPED BEFORE DISPLAY.       DSPLY011
C**          S = NUMBER OF SPACES FROM LEFT SIDE OF PAGE TO          DSPLY012
C**              BE SKIPPED BEFORE DISPLAY.                          DSPLY013
C**          M = NUMBER POINTS IN EACH SET.                          DSPLY014
C**          N = NUMBER OF SETS OF POINTS.                           DSPLY015
C**          W = WIDTH OF DISPLAY IN PRINT SPACES.                   DSPLY016
C**          LN = LENGTH OF DISPLAY IN PRINT LINES.                  DSPLY017
C**          A = ARRAY OF SINGLE CHARACTERS, DIMENSIONED A(N),TO     DSPLY018
C**              REPRESENT THE TREND FOR EACH SET (EX.- DATA A/1HA,  DSPLY019
C**              1HB,...ETC.)                                        DSPLY020
C**       PLOT = ARRAY OF SINGLE CHARACTERS GENERATED BY GP TO       DSPLY021
C**              DISPLAY TRENDS, DIMENSIONED PLOT (LN,W).            DSPLY022
C**                                                                  DSPLY023
      INTEGER S, W, W1                                               DSPLY024
C                                                                    DSPLY025
      DIMENSION X(M), Y(M,N), A(N), PLOT(LN,W)                       DSPLY026
C                                                                    DSPLY027
      DATA BLANK/1H /, EDGE/1H*/                                     DSPLY028
C                                                                    DSPLY029
C**    CHECK MAXIMUM WIDTH AND LENGTH REQUESTED AND                  DSPLY030
C**    EXIT IF NOT CORRECT                                           DSPLY031
C**                                                                  DSPLY032
      IF (S+W .GT. 131) GO TO 900                                    DSPLY033
      IF (L+LN .GT. 58) GO TO 800                                    DSPLY034
C                                                                    DSPLY035
C**    FIND MINIMUM AND MAXIMUM OF X AND Y                           DSPLY036
C**                                                                  DSPLY037
      XMAX=X(1)                                                      DSPLY038
      XMIN=X(1)                                                      DSPLY039
C                                                                    DSPLY040
      DO 10 I=2,M                                                    DSPLY041
      IF (X(I) .GT. XMAX) XMAX=X(I)                                  DSPLY042
   10 IF (X(I) .LT. XMIN) XMIN=X(I)                                  DSPLY043
C                                                                    DSPLY044
      YMAX=Y(1,1)                                                    DSPLY045
      YMIN=Y(1,1)                                                    DSPLY046
C                                                                    DSPLY047
      DO 20 I=1,M                                                    DSPLY048
      DO 20 J=1,N                                                    DSPLY049
      IF (Y(I,J) .GT. YMAX) YMAX=Y(I,J)                              DSPLY050
   20 IF (Y(I,J) .LT. YMIN) YMIN=Y(I,J)                              DSPLY051
C                                                                    DSPLY052
C**    COMPUTE SCALE FACTOR -- P FOR X, Q FOR Y                      DSPLY053
C**                                                                  DSPLY054
      P=FLOAT(M-1)/(XMAX-XMIN)                                       DSPLY055
```

```
      Q=FLOAT(LN-1)/(YMAX-YMIN)                                     DSPLY056
C                                                                   DSPLY057
C**   BLANK PLOT ARRAY                                              DSPLY058
C**                                                                 DSPLY059
      DO 30  I=1,W                                                  DSPLY060
      DO 30  J=1,LN                                                 DSPLY061
   30 PLOT(J,I)=BLANK                                               DSPLY062
C                                                                   DSPLY063
C**   CONSTRUCT BORDER OF DISPLAY                                   DSPLY064
C**                                                                 DSPLY065
      DO 40 J=1,LN                                                  DSPLY066
      I=1                                                           DSPLY067
      PLOT(J,I)=EDGE                                                DSPLY068
      I=W                                                           DSPLY069
   40 PLOT(J,I)=EDGE                                                DSPLY070
C                                                                   DSPLY071
      W1=W-1                                                        DSPLY072
      DO 50 I=2,W1                                                  DSPLY073
      J=1                                                           DSPLY074
      PLOT(J,I)=EDGE                                                DSPLY075
      J=LN                                                          DSPLY076
   50 PLOT(J,I)=EDGE                                                DSPLY077
C                                                                   DSPLY078
C**   COMPUTE SUBSCRIPTS AND INSERT TREND CHARACTER IN             DSPLY079
C**   PLOT ARRAY                                                    DSPLY080
C**                                                                 DSPLY081
      DO 60 I=1,M                                                   DSPLY082
      DO 60 J=1,N                                                   DSPLY083
      I1=1+INT(0.5+P*(X(I)-XMIN))                                   DSPLY084
      J1=LN-INT(0.5+Q*(Y(I,J)-YMIN))                                DSPLY085
   60 PLOT(J1,I1)=A(J)                                              DSPLY086
C                                                                   DSPLY087
C**   SKIP L LINES BEFORE BEGINNING DISPLAY PRINTING               DSPLY088
C                                                                   DSPLY089
      DO 70 K=1,L                                                   DSPLY090
   70 WRITE (6,600)                                                 DSPLY091
  600 FORMAT (1H )                                                  DSPLY092
C                                                                   DSPLY093
C**   WRITE OUT PLOT ARRAY, SKIPPING S SPACES BEFORE PRINTING      DSPLY094
C**   EACH LINE OF DISPLAY                                          DSPLY095
C**                                                                 DSPLY096
      DO 80 J=1,LN                                                  DSPLY097
   80 WRITE(6,601) (BLANK,K=1,S),(PLOT(J,I),I=1,W)                  DSPLY098
  601 FORMAT (132A1)                                                DSPLY099
      WRITE (6,602) XMIN,XMAX,YMIN,YMAX                             DSPLY100
  602 FORMAT (1H0,5X,6HXMIN =E16.8,10X,6HXMAX =E16.8,10X,          DSPLY101
     X       6HYMIN =E16.8,10X,6HYMAX =E16.8)                       DSPLY102
      RETURN                                                        DSPLY103
C**                                                                 DSPLY104
C**   ERROR MESSAGES BEFORE TERMINATION                            DSPLY105
C**                                                                 DSPLY106
  800 WRITE (6,603) L, LN                                           DSPLY107
  603 FORMAT (30HAL+LN IS GREATER THAN 58     L =I3,5X,4HLN =I3)    DSPLY108
      CALL FXEM                                                     DSPLY109
  900 WRITE (6,604) S, W                                            DSPLY110
  604 FORMAT (30HAS+W IS GREATER THAN 131     S =I3,5X,3HW =I3)     DSPLY111

      CALL FXEM                                                     DSPLY112
      STOP                                                          DSPLY113
      END                                                           DSPLY114
```

SUBROUTINE DATA

```
      SUBROUTINE DATA
      RETURN
      END
```

APPENDIX IV

STATISTICAL FORMULAS

Mean (Arithmetic Average) - M

$$M_j = \sum_{i=1}^{n_j} \frac{X_{ij}}{n_j} ,$$

where

X_{ij} = each individual value (SN) in a column "j"

n_j = total number of X_{ij} in a given column "j"

Standard Deviation - SD

$$SD_j = \sqrt{\frac{\sum_{i=1}^{n_j} (X_{ij} - M_j)^2}{n_j - 1}}$$

SD is a measure of dispersion ("scatter") of a given column of data.

Coefficient of Variation - CV

$$CV_j = 100 \times \frac{SD_j}{M_j}$$

Since CV is a calculation of dispersion per unit mean, it is an excellent indicator of precision. Minimization of CV is the goal.

Product-Moment Correlation Coefficient - r

$$r_{jk} = \frac{(\frac{1}{n-1}) \sum_{i=1}^{n} (X_{ij} - M_j)(X_{ik} - M_k)}{(SD_j)(SD_k)}$$

r between any two columns of data "j" and "k."

$n = n_j = n_k$ = total number of data points per column.

This "r" is often called the sample correlation coefficient. This can be related to "ρ," the population correlation coefficient, as a function of sample size n for given confidence limits, by using standard graphs (Reference 6).

The theory and derivation of these quantities can be obtained from most textbooks on engineering statistics, including Reference 9.

APPENDIX V

IMPROVING SMOKE MEASUREMENT RESULTS TAKEN
WITH MILLIPOᵧ MEDIUM

As explained in Appendix II, the smoke measurement (SN) is a function of Q, the sample size per unit filter media area (scf/sq in).

All other factors being constant, Q is proportional to the amount of particulate matter per unit filter media area, "W" (micrograms/sq in). Also, by definition, SN is a function of the absolute reflectance of the smoke spot.

As also explained in Appendix II, loading curves for given filter medium are graphical relationships of spot reflectance (or optical density) versus W. Since these are exactly analogous to SN vs Q functions, this author spent several hours working with Millipore and Whatman media loading curves produced by GE in an attempt to find a way of improving the results that would be obtained with Millipore media. "Improving" in this case means increasing the average SN of Millipore to the same level produced by Whatman medium (the analysis conclusively demonstrated that Millipore produced results consistently lower than those obtained with Whatman medium).

The SN from Whatman and Millipore media are reported at Q values of 0.300 and 0.0565 scf/sq in, respectively. Since SN varies with Q, the problem was to find that value of Q for Millipore medium that produced the same SN as the accepted value of Q for Whatman medium (0.300 scf/sq in). To do this, the author worked with average differences in SN taken from Table I and the GE loading curves.

Such an approach can yield an approximate answer, at best, because the difference between the two loading curves is not constant (one curve is not merely the translation of the other). The curves diverge increasingly with increasing W. Figure 9 demonstrates this.

Only the lower portion of the SN range (about 10 to 50) was used to minimize the effect of this divergence. This is reasonable since SN of about 10 to 50 are of greatest concern (see Section III2).

The final answer was that Millipore with a Q of 0.0650 scf/sq in would give SN of about the same as Whatman with $Q = 0.300$ scf/sq in. It cannot be over-emphasized that this is an approximate answer. Additional experimentation is necessary to corroborate this value.

Even if $Q = 0.0650$ scf/sq in proves to be the "proper" value for use with Millipore medium, it will still be accurate only for part of the range, although fortunately the most important part (SN of 10 to 50). It should also be noted that this adjustment of Q affects the magnitude of resultant SN only; the greater dispersion of Millipore results will probably still be present.

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13. ABSTRACT

This report describes a computerized statistical analysis of test data from engine smoke measurements conducted by the Society of Automotive Engineers Technical Committee E-31. This Committee was organized to develop a reasonably simple, precise, and universally acceptable standard for measuring exhaust smoke from aircraft engines. The analysis indicated that the Committee's test data can be used to arrive at statistically meaningful conclusions about four measuring system parameters. "Whatman No. 4" was found to be superior to "Millipore SM" as a filtering medium in this application. All three reflectometers tested were found to produce equivalent results. White reflectometer background shade was found to have slight superiority over black, yet black (i.e., absolute reflectance less than 5%) was recommended as a safeguard against unknown factors. The lower sampling flow rate (0.0041 scfs) was found to have produced slightly, yet consistently, higher smoke density readings than the higher flow rate (0.0085 scfs) tested.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Aircraft Engine Exhaust Smoke						
Smoke Abatement						
Smoke Measurement						
Filter Media						
Reflectometers						
Isokinetic Sampling						
Statistical Analysis						
Coefficient of Variation						
Product-Moment Correlation Coefficient						
Computerized Data Plots						